A Generalised Procedure for Locating the Optimal Capital Structure

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Abstract: We present here a generalisation of an earlier approach for determining and locating the optimal capital structure of a corporate firm. The approach is the "maximum-value" methodology and the generalisation extends the original problem to one where the firm loses flexibility because of constraints and, therefore, has to move along a specific value-vs-leverage path to get to the optimal capital structure. A practical case of this could, for instance, be that the firm is forced to maintain a constant value while it increases or decreases its leverage. Other types of constraints are also considered and discussed.

1. Introduction

The fundamental concepts that shape modern capital structuring theory were first put together by Modigliani and Miller [M&M] (1958) in a series of propositions. These propositions have, for many years, dominated the thought process by which firms choose their leverage ratio to enhance value. Moreover, the appeal that these concepts have had to academics is enormous, as they are open ended and highly controversial.

A major contribution of M&M's propositions is that they allow one to select, via a formalised process[1], the right balance between debt, equity and assets that raises the overall value of a firm [i.e. firm's value, *FV*]. This increase is brought on by the interest tax shield, which enables *FV* to grow indefinitely with leverage. The negative impact of leverage was later added to demonstrate that *FV* reaches a maximum before it begins to fall, owing to the rising cost of debt overtaking the positive attributes of the tax shield. Thus, what this combination produces is a unique optimal capital structure, where *FV* is at its peak.

Unfortunately, things are not so simple in real life, as evidenced by the large number of studies on how firms generally tend to optimise their capital structure. Although the results are mixed, one prominent

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finding, among others, is that the optimal capital structure is not a unique point, but rather a range of values along the *FV* or weighted average cost of capital [*WACC*] curves [see, e.g., Cal and Ghosh (2003) in addition to others]. This, essentially, implies that it is difficult, if not impossible, for a firm to be able to achieve its optimum by simply inter-changing debt, equity and assets the way the classical M&M methodology dictates. The cause of this is perhaps best attributed to a number of underlying factors and limitations; namely credit, balance sheet constraints and financing decisions (Taggart, 1977), agency costs (Leland, 1998), debt maturity (Berglof and von Thadden, 1994), asset's life (Yi, 2005), firm's size (Hutchinson, 1995), manager's judgement and risk profile (Chu, 1996) and market dynamics (Welch, 2004; Hovakimian, 2004), to name just a few.

With such limitations in place, therefore, it is not surprising that firms typically deviate from M&M's classical framework in their pursuit of the optimal capital structure. An example of this could be that the firm has to maintain a *constant FV* as it changes its leverage[2]. This, for instance, could happen if the firm were forced—i.e. by shareholders, management, etc.—to reduce its leverage by issuing equity to only buy back debt and nothing else. Here, subsequently, the asset size is kept constant while leverage is being changed.

In the other extreme, another approach to reducing leverage is to issue equity to buy assets, while holding the debt level constant. This, in contrast to the above, depicts a growing *FV*, as the firm expands its balance sheet[3] by purchasing assets rather than reducing debt. Note here that the firm is effectively lowering its leverage while holding its debt level constant.

It is clear, therefore, that based on the definition that the optimal capital structure should naturally occur at the point of maximum *FV*, neither of the above situations, which are sketched in Figure 1, appears to possess a well-defined optimum[4]. Nevertheless, it is commonly taken for granted that an optimal capital structure, even under constrained conditions, should indeed exist and it is the objective of this work to try to develop a rational approach for finding it.

We must stress that, for the purpose of locating the optimal capital structure, the literature offers a multitude of works, many of which are conflicting, either with each other or with M&M's original intentions. These works range from being entirely conceptual to all analytical. Bearing in mind the above, the objective here is not to debate which of these are right and which are wrong. Instead, it is to present an alternative approach by modifying a readily available one. This modification comes primarily in the form of (1) tackling the problem from the perspective of constrained optimisation[5] and (2) constructing an effective, yet simple and practical, method for pinpointing the optimal capital structure.

As its framework, the intended approach shall incorporate the "maximum-value" methodology (Cohen, 2004b), mainly because of its relative simplicity and consistency with M&M. This is extended to address cases like the above, whereby the firm operates under constraints brought on by limitations similar to the ones mentioned here.

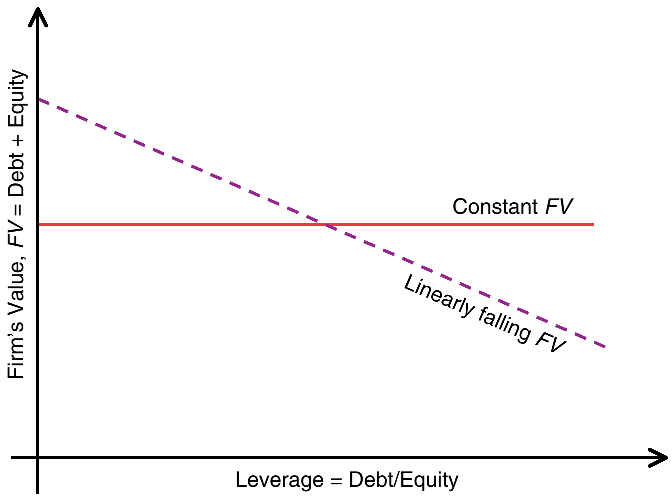

Figure 1: Two examples of constraints imposed on the firm's value, *FV*. The solid line illustrates the case where the firm maintains a constant *FV* and the dashed line depicts a firm that decreases its *FV* linearly with leverage.

Prior, however, for the convenience of the reader, as well as to re-introduce the relevant parameters, we provide an explanation of the original approach, describing how it is used to generate the *WACC* or *FV* curves and locate the optimal capital structure.

2. The Maximum-value Approach

This method has been termed "maximum value" (Cohen, 2004b) because it seeks the optimal capital structure in *FV* without having to resort to the *WACC*. The reason for this is that since *WACC* is defined as

$$WACC \equiv \frac{\tilde{e}_b \times (1-T)}{FV} = \frac{\tilde{e}_b \times (1-T)}{E+D} \tag{2.1}$$

where $\tilde{e}_b$ is the expected earnings before interest and tax [i.e. *EBIT*], T is the tax rate and E and D are equity and debt, respectively, then, with $\tilde{e}_b$ and T constant in accordance with M&M, *WACC* is minimised precisely where the quantity $E + D$, or *FV*, is maximised. It thus follows that one could concentrate only on maximising the *FV* instead of minimising the *WACC*, which has the advantage of not having to involve, as additional parameters, the expected *EBIT* or the CAPM-based beta and/or cost of capital, among others. These, arguably, would only add to the subjectivity and number of assumptions.

The maximum-value approach and how it could be implemented to locate the optimal capital structure are explained here by way of example. For this, we refer to a hypothetical corporate[6] entity, whose simplified financial statement appears in Table 1. It should be mentioned that

TABLE 1: SIMPLIFIED FINANCIAL STATEMENT WITH *E* AND *D* EQUAL TO 200 AND 280, RESPECTIVELY. THE RATES OF TAX, *T*, AND INTEREST, R_D, ARE ASSUMED TO BE 40% [I.E. 25.6/64] AND 5.71% [I.E. 16/280], RESPECTIVELY.

Income Statement	
Expected EBIT	80
Gross interest expense	-16.0
EBT	64.0
Tax	-25.6
Net profits	38.4
Balance Sheet	
Assets	480
Shareholders' equity	200
IB debt	280
Total liab. & equity	480

TABLE 2: THE *FV* CURVE BASED ON TABLE 1, AS OBTAINED FROM THE MAXIMUM-VALUE METHOD OUTLINED IN SECTION 2. THE OPTIMAL CAPITAL STRUCTURE CORRESPONDS TO THE MAXIMUM *FV* IN COLUMN 8. THE CURRENT CONDITION, WITH *D* = 280 AND *E* = 200, FROM TABLE 1, IS ALSO HIGHLIGHTED. GENERATING THIS TABLE REQUIRES AN UNDERLYING CREDIT MODEL, WHICH PROVIDES THE SPREAD AS A FUNCTION OF CERTAIN INPUTS. THE RISK-FREE INTEREST RATE, WHICH IN THIS CASE TURNS OUT TO BE 3.76%, IS COMPUTED BY SUBTRACTING THE CALCULATED SPREAD OF 1.95% FROM THE EFFECTIVE INTEREST RATE OF 5.71%, ALL AT THE CURRENT CONDITION.

D (1)	spread (2)	Interest rate (3)	D* (4)	E (5)	ϕ (6)	V* (7)	FV (8)	
0	0.00%	3.76%	0.0	455.2	0.00	455.2	455.2	
40	0.11%	3.87%	41.1	430.5	0.09	471.6	470.5	
80	0.30%	4.06%	86.3	403.4	0.20	489.7	483.4	
120	0.55%	4.31%	137.5	372.7	0.32	510.2	492.7	
160	0.84%	4.61%	195.9	337.7	**0.47**	533.5	**497.7**	Optimal
200	1.18%	4.94%	262.7	297.6	0.67	560.2	497.6	
240	1.55%	5.31%	338.8	251.9	0.95	590.7	491.9	
280	**1.95%**	**5.71%**	**425.3**	**200.0**	**1.40**	**625.3**	**480.0**	Current
320	2.39%	6.15%	522.9	141.5	2.26	664.3	461.5	
360	2.85%	6.61%	632.3	75.8	4.75	708.1	435.8	

(1) Debt increased in increments.
(2) Interest rate spread over the risk-free rate, assumed to be given.
(3) Interest rate equals risk free rate + spread from Column 2.
(4) "Riskless" debt, calculated from Equation 2.3.
(5) Equity, calculated from constant *Vu** using Equation 2.2.
(6) Leverage ratio, ϕ, equals *D/E*, obtained from Columns 1 and 5, respectively.
(7) Riskless firms value, calculated as *D** [from Column 4] plus *E* [from Column 5].
(8) Firm's value, calculated as *D* [column 1] plus *E* [Column 5].

what follows is a concise description of the procedure, as a more detailed one is available elsewhere (Cohen, 2004b).

Table 1 portrays the current position of the firm's financial statement, with $D = 280$ and $E = 200$. The tax rate, T, is assumed to be 40% and the effective interest rate, R_D, is calculated as the gross interest paid divided by the debt, being 16/280, which equals 5.71%.

Next, one needs to generate Table 2, which ultimately yields the *FV* curve and the location of the optimal capital structure as the point of maximum *FV*. Column 1 lists the debt in incremental levels—in this case starting at zero, ending at 360 and moving at increments of 40.

The spreads associated with the different debt levels are then computed in Column 2. For this, a credit model is required and a process similar to that outlined in Section 2.3 of Cohen (2004b) is generally employed to obtain them[7]. Once these are determined, the risk-free rate, R_f, may be extracted using the spread corresponding to the current situation, where $D = 280$ and $E = 200$. In this example, the spread turns out to be 1.95%, which results in an R_f of 5.71% minus 1.95%, or 3.76%. Given R_f, therefore, the effective interest rates are then determined for the different debt levels, as shown in Column 3.

Before moving any further, one needs to calculate the unlevered value of the firm, V_u^*. This, according to M&M, happens to be the fundamental constant that underlies a specific *FV* or *WACC* curve. It can be shown that [see, e.g., Cohen (2001) or other references discussing the M&M theorems]

$$V_u^* = E + (1 - T)D^* = constant \tag{2.2}$$

where D^* is the "risk-less" debt, which is given by (Cohen 2004b)

$$D^* \equiv \frac{R_D}{R_f} D \tag{2.3}$$

It thus turns out that $D^* = (5.71\%/3.76\%) \times 280 = 425.3$ for the current condition depicted in Table 1 and highlighted in Table 2. Substituting this, along with $E = 200$, into Equation 2.2, yields $V_u^* = 200 + (1 - 0.4)425.3 = 455.2$ as the unlevered value of the firm in question.

The fact that $V_u^* = constant$ underlies the FV curve presented in Table 2 allows the equity, E, in Column 5 to be computed at each level of debt. For instance, at $D = 0$, where D^* also equals zero, we get $E = V_u^* = 455.2$, and where, for instance, $D = 120$ and $D^* = (4.31\%/3.76\%) \times 120 = 137.5$, according to 2.3 above, E turns out to be $455.2 - (1 - 0.4)137.5 = 372.7$.

With E and D^* known at each level of D, the leverage, ϕ, the "risk-less" value of the firm, V^* and the levered firm's value, FV, may finally be determined via the following identities:

$$\phi \equiv \frac{D}{E} \tag{2.4a}$$

$$V^* \equiv D^* + E \tag{2.4b}$$

and

$$FV \equiv D + E \tag{2.4c}$$

Applying the above procedure to all the different debt levels, therefore, leads to the FV curve depicted in Figure 2, which, for this particular example, acquires an optimal capital structure at around D, E and ϕ of 160, 337.7 and 0.47, respectively.

An important aspect of this method is that, in order to achieve its optimal capital structure, a firm requires considerable flexibility to be able to freely adjust its assets and liabilities on the balance sheet. To illustrate, let us refer again to Table 2, where the total debt [Column 1] and equity [Column 5] are 120 and 372.7, respectively. The calculations indicate that raising the debt from 120 to 160 reduces the equity from 372.7 to 337.7 via share buyback. The question, therefore, is with an added incremental debt of 40 and an equity purchase of only 35, where does the remaining 5 go? The answer is that it goes into funding the purchase of additional assets, which explains the subsequent rise in the total value of the firm by an amount of 5—i.e. from 492.7 to 497.7.

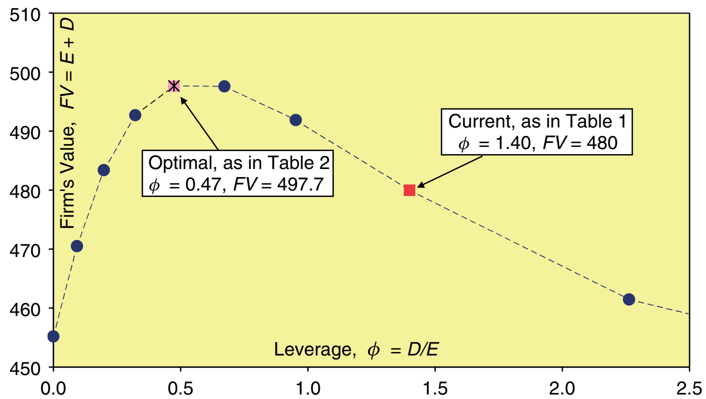

Figure 2: Firm's value, FV, plotted against leverage, ϕ, as obtained from the method outlined in Section 2, using the results in Tables 1 and 2.

The above then leads to the next question of what happens if, for example, suitable assets were not available and, instead, the firm was compelled to purchase an amount of 40 in equity so as to exactly match the incremental debt raised[8]? This, by all means, could be done, but the resulting point will lie on a different $WACC$ or FV curve.

The point raised here, which indirectly alludes to the limitations mentioned earlier, poses the additional issue of how does all this impact a firm's choice of the optimal capital structure, knowing that operating under constraints will tend to force the entity to deviate from the classical M&M framework as it targets optimality? To answer, one must first put this deviation into perspective, then understand and exploit it to generate a useful, yet practical, tool for identifying and locating the optimal capital structure in the presence of constraints. This is addressed in the next section.

3. A Generalisation of the Maximum-value Method

Clearly, the maximum-value method for constructing the FV curve, as outlined in Section 2, is nothing but a constrained-optimisation problem, whereby one is maximising the value of FV subject to the constraint imposed by Equation 2.2. What this means is that the whole of the FV curve, which is generated in the way described above, derives itself from a single value of V_u^*—namely 455.2, as calculated. Therefore, it is purely by construction that each and every point, including the current and optimal, which goes into shaping the curve in Figure 2, originates from the same V_u^*.

So what are the implications of this on the firm's flexibility when it comes to adjusting its balance sheet, as discussed in the last paragraph of Section 2? For instance, what would happen if this freedom were not there and, instead, the firm was forced to maintain a *constant FV*, instead of V_u^*, in its attempt to optimise its capital structure through a direct exchange of debt for equity, or vice versa[9]? Would the same, exact method of optimising the capital structure still hold? Obviously not, because by holding FV constant, $WACC$ also stays unchanged, as per Equation 2.1[10], and, therefore, based on the classical definition of $WACC$, there would be no optimal capital structure in sight at any leverage.

To get around this, the fundamental approach to determining the optimal capital structure must be modified altogether by placing the constraint on FV rather than on V_u^*. The next section illustrates how this could be attained.

3.1 The Case of Constant *FV* as Constraint

Let us refer to Figure 3 to explain the situation. This graph, which contains several lines, has FV plotted against leverage, ϕ. The horizontal line depicts an FV that is held constant at 480, again equal to that in Table 1. The other four lines, designated by the numbers 1 through 4, illustrate, in contrast, various FV curves, each originating from a different unlevered value, V_u^*, numbered V_{u1}^*, V_{u2}^*, V_{u3}^* and V_{u4}^*, respectively, with Lines 2,

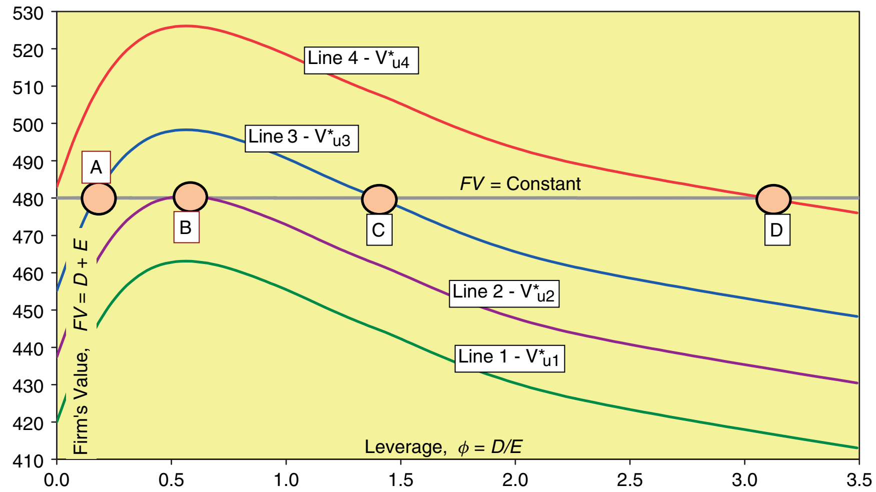

Figure 3: Illustration of how a constant *FV* curve can be constructed from points belonging to separate *FV* curves, each based on a different unlevered value, V_u^*.

3 and 4 intersecting the constant *FV*. Each of these, accordingly, is a by-product of the maximum-value method. Note that Line 3 corresponds to the *FV* curve derived in Table 2, which has a $V_u^* = V_{u3}^* = 455.2$ and a maximum *FV* of 497.7 at $\phi = 0.47$. The other lines possess different values for V_u^* and *FV*.

Therefore, what Figure 3 reveals is that the constrained *FV* curve, which in this particular case is constant in leverage, can be constructed from points that belong to other *FV* curves, each associated with its own characteristic V_u^*.[11] Here, for instance, among the infinite number of points that go into producing the constant *FV* line, we have highlighted Points A to D, which pertain to the intersecting lines, 2, 3 and 4. Note that while Points A and C belong to Line 3, Point B is associated with Line 2 and Point D with Line 4. Clearly, therefore, any *FV* curve, constrained or otherwise, is, technically speaking, the locus of an infinite number of points, each originating from an *FV* curve prescribed by an unlevered value, V_u^*, which is specific to it.[12]

Thus, how does one determine the location of the optimal capital structure in the case presented in Figure 3, where *FV* is held constant, or subjected to another constraint? For this, a modification to the definition of the optimal capital structure, as per M&M, becomes necessary. This is stated below in the form of a proposition:

Proposition: *The optimal capital structure of a firm lies where the difference between the levered value, FV, and its local, unlevered counterpart, V_u^*, is maximised.*

Observe that the above differs from the classical definition, where emphasis is placed on maximising *FV* irrespective of V_u^*. Notwithstanding, the proposition is fully consistent with the maximum-value methodology, where the focus lies on maximising *FV* subject to constant V_u^*. Let us now go to Table 3 for a step-by-step description of the procedure.

The analysis here incorporates two earlier identities, namely 2.4a and 2.4c. Combining these, one obtains the relations for D and E in terms of *FV* and ϕ as:

$$D = \frac{\phi}{1+\phi} \times FV \tag{3.1}$$

and

$$E = \frac{1}{1+\phi} \times FV \tag{3.2}$$

Column 1 in Table 3 comprises the leverage, ϕ, in this case ranging between 0 and 2.2, and rising in increments of 0.2. Column 2 displays the value of the debt appropriate to the different levels of ϕ, as derived from Equation 3.1 using $FV = constant = 480$ [from Table 1] as the constraint. This assumes that in the process of substituting debt for equity, or vice versa, the firm is restricted to holding *FV* constant at 480.[13]

The value of equity, E, is worked out next in Column 3, the calculation of which is based on Equation 3.2 and $FV = constant = 480$. Summing D and E from Columns 2 and 3, respectively, leads to Column 4, which confirms the forced constraint on *FV*. The spread is displayed in Column 5. Column 6 depicts the effective interest rate, R_D, which, as before, is the sum of the risk-free rate and spread. The methods for finding the risk-free rate and spread are explained in Section 2.

Column 7 consists of the risk-less debt, which, according to Equation 2.3, comes from dividing the product of Columns 2 and 6 by the risk-free rate. The "local" unlevered firm's value, V_u^*—that which corresponds to a specific constrained *FV* point and which may not be constant along the *constrained* curve, as demonstrated in Figure 3—is displayed in Column 8. This is determined by employing Equation 2.2, given $T = 40\%$, as assumed, and E and D^* from Columns 3 and 7, respectively. Finally, Column 9 depicts the difference between *FV* and the local V_u^*. Here, as well as in Figure 4, which plots both *FV* and the difference $FV - V_u^*$ as functions of ϕ, we observe that the optimal capital structure—the point where this difference is maximal—exists at a leverage of around 0.6. This contrasts to the example in Table 2, where the optimal is found to occur at a different leverage ratio. The above, therefore, presents a simple demonstration of how constraining a firm's path along the leverage curve[14]—as it shifts its debt, equity and assets in the balance sheet—could affect the location of the optimal capital structure.

3.2 The Case of Linear *FV* as Constraint

Another example of a constrained *FV* may involve a firm whose value is forced to vary linearly with leverage, i.e.

$$FV = a + b\phi \tag{3.3}$$

where a and b are constants. Once again, the procedure discussed in Section 3.1, leading to Table 3, could be followed, but instead of a constant *FV*, we substitute Equation 3.3 [with $a = 445$ and $b = -25$ in this particular instance to signify a linearly falling *FV*] into 3.1 and 3.2 to obtain the values of D and E as functions of ϕ.[15] The result of this is displayed in Table 4, where, once again, the current firm's financial statement has been chosen

TABLE 3: THE *FV* CURVE, AS OBTAINED FROM THE GENERALISED METHOD OF IMPOSING A CONSTRAINT ON *FV*, AS DESCRIBED IN SECTION 3. THE OPTIMAL CAPITAL STRUCTURE CORRESPONDS TO THE MAXIMUM IN THE DIFFERENCE BETWEEN *FV* AND V_U^*, DISPLAYED IN COLUMN 9. THIS TABLE IS BASED ON THE HOLDING *FV* = *CONSTANT*, AS DEPICTED IN COLUMN 4 AND OUTLINED IN SECTION 3.1. THE CURRENT CONDITION OF THE FIRM, WITH *D* = 280 AND *E* = 200 AS IN TABLE 1, IS AGAIN ASSUMED TO HOLD. AS BEFORE, THE CURRENT EFFECTIVE INTEREST RATE IS 5.71% AND THE RISK-FREE RATE 3.76%.

ϕ (1)	D (2)	E (3)	FV (4)	spread (5)	Interest rate (6)	D* (7)	Vu* (8)	FV-Vu* (9)	
0.0	0.0	480.0	480	0.00%	3.76%	0.0	480. 0	0.0	
0.2	80.0	400.0	480	0.30%	4.06%	86.3	451.8	28.2	
0.4	137.1	342.9	480	0.67%	4.43%	161.5	439.8	40.2	
0.6	180.0	300.0	480	1.01%	4.77%	228.1	436. 9	**43.1**	Optimal
0.8	213.3	266.7	480	1.30%	5.06%	287.0	438.8	41.2	
1.0	240.0	240.0	480	1.55%	5.31%	338.8	443.3	36.7	
1.2	261.8	218.2	480	1.77%	5.53%	384.7	449.0	31.0	
1.4	**280.0**	**200.0**	**480**	**1.95%**	**5.71%**	**425.3**	**455.2**	**24.8**	Current
1.6	295.4	184.6	480	2.12%	5.88%	461.5	461.5	18.5	
1.8	308.6	171.4	480	2.26%	6.02%	493.8	467.7	12.3	
2.0	320.0	160.0	480	2.39%	6.15%	522.9	473. 7	6.3	
2.2	330.0	150.0	480	2.50%	6.26%	549.1	479. 5	0.5	

(1) Leverage increasing in increments of 0.2.
(2) Debt calculated from Equation 3.1.
(3) Equity obtained from Equation 3.2.
(4) Firm's value, calculated by adding Columns 2 and 3.
(5) Spread obtained from some credit-rating methodology, as described in Section 2.
(6) Effective interest rate obtained from adding the risk-free rate and spread from Column 5.
(7) The riskless debt computed using Equation 2.3 [product of Columns 2 and 6 divided by the risk-free rate].
(8) The unlevered value of the firm, obtained from Equation 2.2.
(9) The difference between *FV* [Column 4] and *Vu** [Column 8].

to correspond to that in Table 1.[16] There is no need here to explain the contents of the various columns, as the process is identical to that already discussed in the preceding section, as well as documented at the bottom of Table 3.

4. Summary and Conclusions

This work extends the original one on finding the optimal capital structure of a corporate firm (Cohen, 2004b) by setting the constraint on the firm's value rather than on its unlevered value. The current approach, therefore, is a generalisation of the earlier one, taking into account the typical limitations that may be imposed on a firm—i.e. its freedom to juggle the balance sheet in pursuit of the optimum. These limitations, which are brought on by real-life circumstances, strongly reduce the flexibility that a corporate entity has in hand to exchange debt for equity and assets, and vice versa.

It is observed that incorporating these constraints into the original problem requires a modification of how one characterises the location of the optimum. Whereas in the original situation the optimum was identified as the region where *FV* is maximised, the present one describes it as where the difference between the firm's value, *FV*, and the *local* unlevered value, V_u^*, is maximised. Notwithstanding this and considering that in the earlier approach V_u^* remains constant along the *FV* curve, it is clear that this new characterisation is fully consistent and, therefore, applicable to both, the original method and its extension presented here.

TABLE 4: THE *FV* CURVE, AS OBTAINED FROM THE GENERALISED METHOD OF IMPOSING A CONSTRAINT ON *FV*, AS DESCRIBED IN SECTION 3. THE OPTIMAL CAPITAL STRUCTURE CORRESPONDS TO THE MAXIMUM IN THE DIFFERENCE BETWEEN *FV* AND V_U^*, DISPLAYED IN COLUMN 9. THIS TABLE IS BASED ON AN *FV* THAT FALLS LINEARLY WITH LEVERAGE, ϕ, AS DEPICTED IN COLUMN 4 AND OUTLINED IN SECTION 3.2. THE CURRENT CONDITION OF THE FIRM, WITH D = 280 AND E = 200 AS IN TABLE 1, IS AGAIN ASSUMED TO HOLD. AS BEFORE, THE CURRENT EFFECTIVE INTEREST RATE IS 5.71% AND THE RISK-FREE RATE 3.76%.

ϕ (1)	D (2)	E (3)	FV (4)	spread (5)	Interest rate (6)	D* (7)	Vu* (8)	FV-Vu* (9)	
0.0	0.0	515.0	515	0.00%	3.76%	0.0	515.0	0.0	
0.2	85.0	425.0	510	0.33%	4.09%	92.4	480.4	29.6	
0.4	144.3	360.7	505	0.72%	4.48%	172.0	463.9	41.1	
0.6	187.5	312.5	500	1.07%	4.83%	240.8	457.0	**43.0**	**Optimal**
0.8	220.0	275.0	495	1.36%	5.12%	299.5	454.7	40.3	
1.0	245.0	245.0	490	1.60%	5.36%	349.1	454.4	35.6	
1.2	264.5	220.5	485	1.79%	5.55%	390.6	454.8	30.2	
1.4	**280.0**	**200.0**	**480**	**1.95%**	**5.71%**	**425.3**	**455.2**	**24.8**	**Current**
1.6	292.3	182.7	475	2.08%	5.84%	454.1	455.2	19.8	
1.8	302.1	167.9	470	2.19%	5.95%	477.9	454.6	15.4	
2.0	310.0	155.0	465	2.27%	6.04%	497.4	453.4	11.6	
2.2	316.3	143.8	460	2.34%	6.11%	513.2	451.7	8.3	

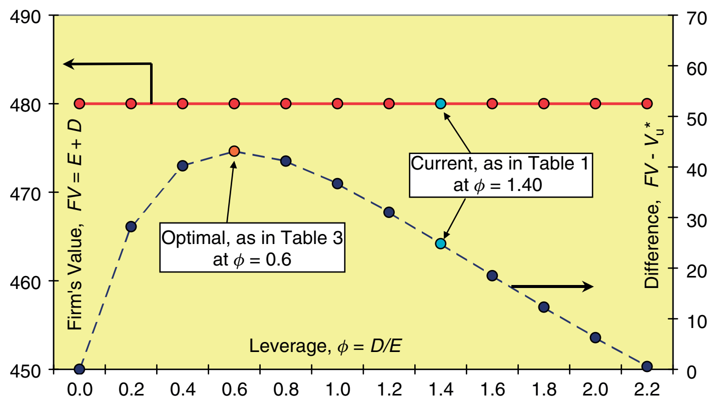

Figure 4: The result of Table 3, portraying how a constrained *FV*, which is maintained at a constant [at 480], could possess an optimal capital structure, which lies where the difference between *FV* and V_u^* is maximised.

FOOTNOTES & REFERENCES

1. The basic idea behind M&M's process is that the firm's value could be generated separately from either the balance sheet [as equity + debt] or the income statement [as the present value of cash flow]. The two valuations must, of course, match and M&M's propositions provide the means for their reconciliation.
2. Leverage is defined here as the debt-to-equity ratio.
3. It should, of course, be understood that as the balance sheet grows, the income statement is also affected in a manner consistent with the changes in the balance sheet (see Footnote 4).
4. Although the case of the linearly falling *FV* has a maximum at a leverage of zero, it is not the optimal capital structure since the tax rate may be different from zero.
5. The constraint may be a result of any of the limitations mentioned earlier.
6. A major distinction between a corporate and a financial entity is that the *EBIT* of the latter is not constant, as assumed by M&M for corporates, but varies with the size of the balance sheet [see, e.g. Cohen (2004a) or references therein].
7. For our purposes here, and for the reason that determining the spreads is out of the scope of this work, we shall take these as given. Obviously, the final results depend on the spread curve, although it must be stressed that the general trends are expected to remain the same as long as these spreads rise with the probability of default.
8. Note that this leads to the constant-*FV* constraint described earlier.
9. It is noted that this type of constraint, namely *FV = constant* as a function of leverage, seems to be one of the most commonly studied, as attested by the extensive literature related to this area [see, e.g., Damodaran (1994) and many others that have preceded and followed it].
10. This assumes that the *EBIT* and tax rate, *T*, also remain unchanged. See Footnote 15 for a discussion.
11. Refer to the paragraph before the last in Section 2.
12. It is important to discuss how the *EBIT* and *T* are presumed to change in this plane. A common assumption underlying the constant *FV* line is that both *EBIT* and *T* are constant along it. As these two parameters are also assumed to remain constant along the intersecting lines of constant V_u^*, it follows, therefore, that all the intersecting lines will also acquire the same *EBIT* and *T* as those on the *FV = constant* line.
13. This is accomplished by a simple one-to-one exchange between debt and equity.
14. This path is reflected by the constraint imposed on the *FV* as the firm varies its leverage.
15. As mentioned earlier, a falling *FV* with increasing leverage could result from issuing equity to purchase assets while holding debt constant.
16. This was achieved by letting α in Equation 3.3 be equal to 445.

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