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The energy level of size distribution probability functions

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ABSTRACT

The size distribution probability (SDP) is, among other functions, an important fingerprint of dispersions. Here we demonstrate that, associated with every size distribution function, there is an energy level, which could as well serve as the system's fingerprint. An advantage of using the energy level in conjunction with the SDP function is that, in addition to quantifying the 'energy' of the distribution, one can easily determine whether a proposed theoretical SDP function can exist or not.

§ 1. INTRODUCTION

Representing systems of particles by their size distribution probability (SDP) functions is common practice in many science and engineering fields, ranging from environmental studies of particulate-caused pollution to the applications of powder processing and technology in materials science and engineering. As a result, a number of theoretical SDP functions have been proposed over the years to account for the different types of distributions observed.

It was recently shown that theoretical SDP functions might be subject to entropic limitations, which could otherwise render them useless in practice (Cohen 1993). Our objectives here are firstly to introduce and derive, with the aid of concepts from statistical mechanics, an energy-related property of such functions, and secondly to demonstrate its uses in quantifying the 'energetics' of some typical cases, and in assessing their suitability and limitations in representing actual particle-size distributions.

§ 2. PROBLEM FORMULATION

Consider a system consisting of a total of N_0 primary units or building blocks. These can be arranged into clusters of different sizes, satisfying at any instant in time the following relations:

$$N_0 = \sum_i iN_i, \tag{1}$$

and

$$N = \sum_i N_i, \tag{2}$$

where N is the total number of clusters, i is the cluster size and N_i is the number of clusters of size i. In typical situations, N_0 remains constant (i.e. total mass is conserved), while N_i and N may or may not vary with time.