Development of the Cluster-Size Distribution in Flowing Suspensions

Ruben D. Cohen
Dept. of Mechanical Engineering and Materials Science, Rice University, Houston, TX 77251

The complexities involved in modeling the simultaneous formation and disintegration of clusters or aggregates in suspensions are well documented in the literature. It is known that breakup can occur in conjunction with aggregation once the suspension is mechanically stirred or undergoes continuous flow, even during sedimentation. Works related to the microscopic nature of aggregation and breakup in such cases have typically centered on detailed analyses of the motion of individual particles or clusters as they approach other particles or clusters to form larger ones—a phenomenon observed in sedimentation, shear, and turbulent flows.

Macroscopically, the problem constitutes an overall study of the development of the cluster-size distribution by simultaneous coagulation and breakup. Typical analytical approaches to such have been through the population balance equation (for example, Elminyawi et al., 1991) and statistical means (Cohen, 1990, 1991a). While the generality of the former method must be appreciated, it is recognized that obtaining solutions requires inserting certain empirical parameters related to the rates of breakup and coagulation, in addition to other unknowns, into the kernels of the integro-differential equation. On the other hand, the latter approach assumes a purely random process, which allows it to bypass the need for these assumptions. One, however, would expect this approach to become valid when certain interactions, such as DLVO (based on a theory by Derjaguin, Landau, Verwey, and Overbeek), are overwhelmed—for example, by exposing the suspension to rapid mechanical mixing, as discussed briefly in Cohen (1992) and/or when the particles are relatively massive.

There exist many publications dealing with the development of aggregates in closed systems. Upon incorporating inflow and outflow, which would subsequently classify the system as open or continuous flow, significant complications in predicting the cluster-size distributions arise. Once again, typical analytical methods have utilized the population balance equation, an example of which was used by McGrady and Ziff (1988) for the special case of pure fragmentation.

Clearly, all processes involving simultaneous formation and disintegration of clusters possess, at any given time, both stochastic and "orderly" characteristics. If these processes were totally random, then for practical purposes, a single population balance equation, having a universal set of coefficients and kernels, would suffice to solve all problems of this nature. Nevertheless, knowing that the two behaviors (order and disorder) can co-exist, it would be useful to draw a fine line differentiating between the two, since only then the mechanisms at work might be identifiable. As of yet, except for a handful of investigations that provide lengthy descriptions based on visual observations, none has aimed at achieving this goal. A quantitative approach, therefore, could present an important step toward a better understanding of the fundamentals involved in the simultaneous coagulation and breakup of clusters.

This work focuses on two objectives. First, it is to prove that the statistical model proposed by Cohen (1990, 1991a), which is based on closed (or batch) systems, can be extended to open-flow systems, provided that the coagulation-breakup process is random. Second, it is to relate the theoretical predictions with the experimental data obtained from some open-flow experiments to demonstrate the co-existence of stochastic and orderly behaviors, as well as to quantitatively pinpoint and explore their regimes of manifestation.

Problem Formulation

The details of the *evolution* of the cluster-size distribution by simultaneous and random coagulation and breakup in batch systems are discussed by Cohen (1991a). Of importance is the result that

$$p(i) = \frac{1}{e^{\mu} - 1} \frac{\mu^i}{i!} \tag{1}$$

where i is the aggregate size, $p(i)$ is the size-distribution probability defined as:

$$p(i) \equiv \frac{N_i}{N} \tag{2a}$$