An Analytical Process for Generating the *WACC* Curve and Locating the Optimal Capital Structure[*,†]

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Abstract
We present here an analytical process for generating the firm's value [FV] and the weighted-average cost of capital [WACC] curves, with intent to locate the optimal capital structure. The method takes into consideration the relationship between debt, equity and taxes, and places emphasis on the effects of default risk, as well as on the assumptions that underlie the curves. In relation to the proposed approach, it is shown that the conventional one, which is used more commonly in practice, is flawed.

1 Introduction

Capital structuring and, in particular, locating the optimal capital structure, have, for a long time, been a focus of attention in many academic and financial institutions that probe into this area. Academically, the problem is appealing because it is fairly open ended and subject to controversies and criticisms. And, practically, there is great interest, especially in the areas of corporate and project finance, as well as in structured products, as there is a lot of money to be made advising firms on how to improve their capital structure.

The major breakthrough in capital structuring theory came with Modigliani and Miller's [M&M] propositions (Modigliani and Miller, 1958). These, not unexpectedly, have led to a considerable amount of literature, both theoretical and practical, on how to determine and locate the optimal capital structure. Overall, the approaches range from being purely subjective to all analytical, with the former comprising qualitative descriptions of roughly where the optimum should fall and the latter providing comprehensive graphs of the *WACC* and/or *FV*, which depict the location of the optimum.

This work leans more towards the analytical side, focusing on providing a detailed discussion and derivation of the process that leads to the above-mentioned curves. Before going any further, it would be helpful to touch on some of the relevant works and their relation to our objectives.

There are at least two issues that surround the *WACC* and/or the *WACC* curve. Firstly, how accurately can the *WACC* be calculated? And, secondly, if it were at all possible to obtain the *WACC* accurately, would it ever serve as a useful metric to appraise a company's performance? These two issues are covered well in the literature, but since we are interested only in the first, we shall ignore the second, as it has no relevance to our objectives here.[1]

As far as we are aware, there is one conventional technique for obtaining the *WACC* curve. The method revolves heavily around the M&M principles, and makes use of the CAPM-based cost of [or return on] equity[2], the cost of debt, the tax rate and values of debt and equity.

Although practical and theoretical courses[3] abound on how to apply the technique, and its variants, to obtain the curves of interest, the literature, in large, appears to lack any thorough description, detailing the actual procedure itself. We intend here to fill in the gap by providing an

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[†] I express these views as an individual, not as a representative of companies with which I am connected.

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in-depth, step-by-step explanation of how the *FV* and *WACC* curves could be generated. In due course, we will, as well, stumble on a major flaw that the current methodology contains and propose a way to get around it. The final outcome, therefore, shall comprise a process that is not only consistent with M&M's principles, but also more robust than the conventional.

2 Preliminary Background, Definitions and Assumptions

An important implication of the M&M capital structuring theorems is that when there are taxes, debt-related tax benefits, in terms of interest tax shields, accrue, which add value to the firm. The notion itself is surprisingly straightforward and could be presented as (Ross *et al*, 1998)

$$\Delta V = DT \tag{1}$$

where T is the tax rate and ΔV is the incremental value added by taking on a debt of D. The product DT is simply the present value of the interest tax shield.

In view of the above, it is possible to demonstrate numerically [see, e.g., Cohen (2001) for a simple illustration] that debt and equity are coupled to each other, as well as to the firm's unlevered value, via the following relationship:

$$E + (1 - T)D = V_u \tag{2}$$

where E is the equity and V_u the unlevered value. The latter comprises the fundamental constant that goes into producing the *WACC* curve.[4] Putting Equations 1 and 2 together, therefore, leads to the classical relationship:

$$V_L = V_u + DT \tag{3}$$

where V_L is the value, i.e. $E + D$, of the levered firm.

A step-by-step example was then worked out on how the *WACC* curve evolves with increasing debt. For convenience, we have extended that example in Table 1 here to cover a wider range of debt, and to produce more continuous curves of the *WACC* and return on equity [*ROE*] as functions of the leverage. These results are depicted in Figures 2a and 2b, the latter expanding on the *WACC* curve. Finally, Figure 3 displays the behaviour of the *FV*, i.e. $E + D$, as the leverage, D/E, increases.

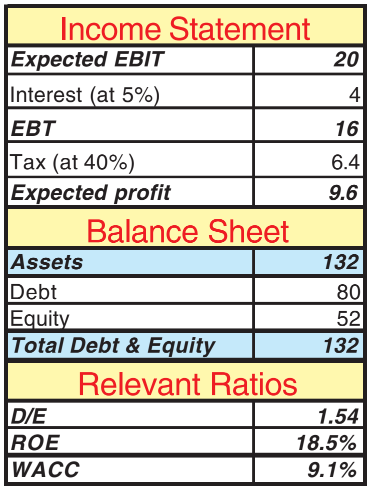
In Figure 2b, we observe the classical shape of the *WACC* curve in a world with no default risk. In the presence of such risks, however, the curve may assume a minimum at some point. It is this minimum that captures most, if not all, of the attention of academics and practitioners who delve into this area.

Our objective here is to explain how one could locate the optimal capital structure, if it exists. In the course of the derivation, we will, as well, demonstrate that the conventional way for deriving the *WACC* curve is flawed.

We shall, never the less, continue to work with the simple case shown in Figure 1, even though actual financial statements tend to be much more complicated. The need to simplify stems from the necessity to shed light on the fundamentals that underlie capital structuring and its optimisation. Without properly understanding these, it is virtually useless and meaningless to pursue more complicated cases.

TABLE 1: CALCULATION OF THE VARIABLES *E*, V_L, *D/E*, *ROE* AND *WACC* VIA APPLYING EQUATIONS 2, 3, 4C AND 4D TO THE FINANCIAL STATEMENT IN FIGURE 1

D	E	V	D/E	ROE	WACC
0	100	100	0.00	12.0%	12.0%
10	94	104	0.11	12.4%	11.5%
20	88	108	0.23	13.0%	11.1%
30	82	112	0.37	13.5%	10.7%
40	76	116	0.53	14.2%	10.3%
50	70	120	0.71	15.0%	10.0%
60	64	124	0.94	15.9%	9.7%
70	58	128	1.21	17.1%	9.4%
80	52	132	1.54	18.5%	9.1%
90	46	136	1.96	20.2%	8.8%
100	40	140	2.50	22.5%	8.6%
110	34	144	3.24	25.6%	8.3%
120	28	148	4.29	30.0%	8.1%
130	22	152	5.91	36.8%	7.9%
140	16	156	8.75	48.8%	7.7%
150	10	160	15.00	75.0%	7.5%



Income Statement	
Expected EBIT	***20***
Interest (at 5%)	4
EBT	***16***
Tax (at 40%)	6.4
Expected profit	***9.6***
Balance Sheet	
Assets	***132***
Debt	80
Equity	52
Total Debt & Equity	***132***
Relevant Ratios	
D/E	***1.54***
ROE	***18.5%***
WACC	***9.1%***

Figure 1: Simplified financial statement, including an income statement and balance sheet

Let us now introduce some of the common variables and terminology, and, also, restate the assumptions that shape the relevant M&M theorems. In due course, we will go through the basic principles that lead

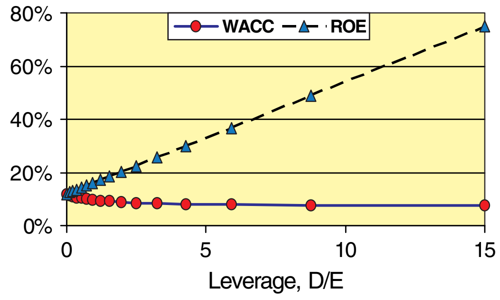


Figure 2a: *WACC* and *ROE* plotted against *D/E*

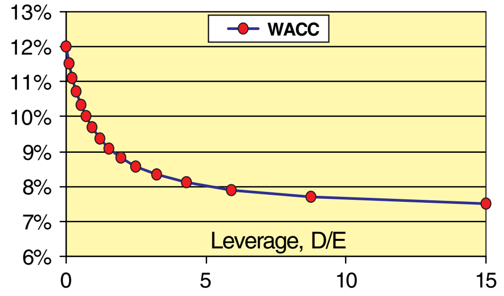


Figure 2b: Expanded view of *WACC versus D/E*

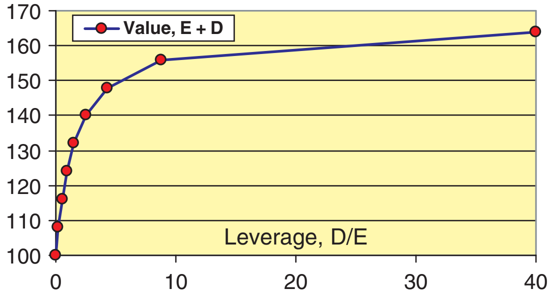


Figure 3: The value of the firm versus its leverage

to the governing equations. The default risk is ultimately brought into the picture, as its impact is demonstrated on certain variables - such as the *WACC*, *ROE*, firm's value, etc. - that play important roles here.

2.1 The Case of Taxes, but with No Default Risks—The "Idealised" Case

We begin here with a financial statement that constitutes the income statement and balance sheet, and, again, focus on the simplified version shown in Figure 1. This depicts a firm that has an asset base or "levered" value, V_L, of 132, a debt, D, of 80 and a balancing equity, E, of 52. In addition, the "expected" *EBIT*[5], $\tilde{e}_b^*$, is taken as 20, which, in turn, is subjected to constant interest and tax rates, R_D^* and T, respectively, of *5%* and *40%*. For simplicity, we assume that the book and market values are the same[6].

These numbers, therefore, lead to an observed leverage, ϕ, of 1.54, an "expected" profit, $\tilde{e}_f^*$, of 9.6, a return on equity, R_E^*, of 18.5% and a *WACC* of 9.1%, all which were obtained using the following definitions:

$$\phi \equiv \frac{D}{E} \tag{4a}$$

$$\tilde{e}_f^* = [\tilde{e}_b^* - R_D^* D] \times (1 - T) \tag{4b}$$

$$R_E^* \equiv \frac{\tilde{e}_f^*}{E} \tag{4c}$$

and, finally

$$WACC^* \equiv \frac{\tilde{e}_b^* \times (1 - T)}{V_L} = \frac{\tilde{e}_b^* \times (1 - T)}{E + D} \tag{4d}$$

Note that the asterisk, which appears in some of the superscripts, implies the hypothetical, idealised case of no default risk.

Let us, in addition, introduce the parameter R_u^* and define it as

$$R_u^* \equiv \frac{\tilde{e}_b^* \times (1 - T)}{V_u} = \frac{\tilde{e}_b^* \times (1 - T)}{E + (1 - T)D} \tag{5}$$

which, in contrast to 4d, is the "unlevered" *WACC*, since it is based on the unlevered value of the firm. On combining Equations 4a-c with 5, we obtain:

$$R_E^* = R_u^* + [R_u^* - R_D^*]\phi(1 - T) \tag{6}$$

which states that, holding R_u^*, R_D^* and T constant, and with $R_u^* > R_D^*$, the cost of equity must rise linearly with leverage. This is M&M's Proposition 2, whose behaviour is clearly displayed in Figure 2a.

2.2 The Role of the Beta

We now discuss how the firm's stock price and its rate of return fit into the picture. CAPM tells us that these enter through the beta[7]of the firm, which, in theory, should depend on leverage - i.e. as leverage increases, so should beta. But what is the relationship?

To find out, we begin with the beta of the levered firm. This beta, which shall be denoted by β_L, is connected to the return on equity, R_E^*, via the definition[8]:

$$\beta_L \equiv \frac{R_E^* - R_D^*}{R_P} \tag{7}$$

where R_p is the market's risk premium. We have, in the absence of default risk[9], taken the risk-free rate, R_f, to be equivalent to the default-risk-free interest rate, R_D^*. If this were not the case, then a slight adjustment to β_L, which accounts for the marginal spread between R_D^* and R_f, would be necessary. However, the proof for this will be left out of here, as is out of the scope of this work.

By setting the leverage, ϕ, equal to zero in Equation 6 and inserting the result in 7, we get the unlevered beta, β_u, as

$$\beta_u \equiv \frac{R_u^* - R_D^*}{R_P} \tag{8}$$

Finally, substituting Equations 7 and 8 into 6 and simplifying, we obtain the well-known relationship between the unlevered and levered betas as

$$\beta_L = \beta_u[1 + \phi(1 - T)] \tag{9}$$

In practice, one would start with the beta of the levered firm, β_L, which we treat here as a given, extract from it β_u using Equation 9, and incrementally increase ϕ to work out the hypothetical levered betas at each stage. For instance, if the firm that is represented by the statement in Figure 1 had a β_L of, let us say, 1.25, and a leverage, ϕ, of 1.54, then its unlevered beta, β_u, would be 0.65. Notwithstanding, in the more realistic world with default risks, it is not that straightforward to obtain the levered beta, as it shall be proven later in more detail. Having now established the notions of the levered and unlevered betas, we shall continue on and bring in the effects of the default risk.

2.3 The Default Risk

It is a fact that every firm is at risk of defaulting on its debts. The extent of this risk is quantifiable by the probability of default. This probability, in turn, leads to the credit rating through a statistical process that incorporates several of the firm's current and expected ratios and other factors. The process itself is too long and involved to be discussed here, but, in all, the rating that ensues helps to provide a common background against which all firms can be compared.

TABLE 2: TYPICAL, STATISTICALLY-OBTAINED RELATIONSHIP BETWEEN RATING AND THE SPREAD OVER THE RISK-FREE RATE. THE RATING CLASSIFICATION HERE IS S&P'S.

rating	spread
C	7.50%
CC	6.00%
CCC	5.00%
B	3.25%
BB	2.00%
BBB	1.50%
A	1.00%
AA	0.50%
AAA	0.20%

The analyst who probes into capital structuring may, never the less, be more interested in the spread over the risk-free rate that this classification system represents. Hence, much statistical work has gone into producing these relations, a sample of which is displayed in Table 2 (Damodaran, 2000).

The overall process of translating debt into rating, then spread and, ultimately, the company's cost of debt, is summarised in Figure 4. Typically, one would start with the interest-bearing debt and a presumed cost of debt, and, together with numbers from the financial statement, compute certain relevant factors and ratios. With the aid of a credit-rating model [CRM], these then lead to an implied rating, which provide us with the spread via, for example, Table 2. The spread, when added to the risk-free rate, yields a new cost of debt[10]. The procedure may also embody a loop, whereby if the calculated cost of debt should differ from the previous one, it is fed back to re-assess the factors and ratios (Damodaran, 2000). The loop continues until the cost of debt converges.

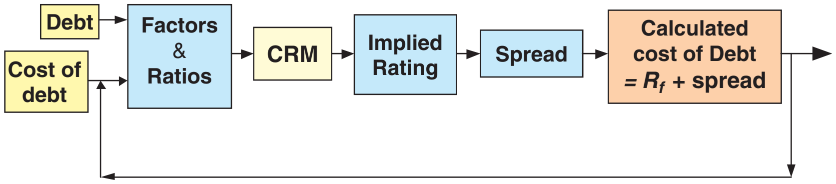


Figure 4: Schematic diagram of the iterative process that converts debt into the cost of debt

Clearly, a full account of the methodology that goes into the above is out of the scope of this work, as it is too lengthy to include here. However, for our purposes, and for the sake of conciseness, we shall treat the course that converts debt, or leverage, into the cost of debt, as shown in Figure 4, as simply a black box, and assume that the relationship between the two is readily known and available. With this in mind, we move on to the next section and discuss the optimisation of the capital structure.

3 Optimisation of the Capital Structure – The Different Approaches

Minimising its cost of capital and/or maximising its value remains the objective of every firm that seeks to optimise its capital structure. Altogether, explanations, both qualitative and analytical, abound on how to optimise the capital structure [e.g. see Chew (1998) and references therein]. Our aim here is similar in that we also try to provide an explanation, albeit quantitative, on how and where this optimum occurs.

Let us return to the classical M&M theorems, which describe a world with taxes but with no default risk, and note the absence of an optimal capital structure in the equations in Section 2.1. If there were one, it should occur at a leverage $\phi \to \infty$. Let us demonstrate.

By definition, the *WACC*, as presented by Equation 4d, may be generalised to account for the default risk as well – i.e.

$$WACC \equiv \frac{\tilde{e}_b \times (1 - T)}{V_L} = \frac{\tilde{e}_b \times (1 - T)}{E + D} \tag{10}$$

where the removal of the asterisks indicates the presence of default risk. Here, theory states that, holding $\tilde{e}_b$, T and V_u constant over any level of debt, D, a rise in debt should correspond to a fall in the *WACC* [recall that $E + D = V_u + DT$ in the absence of default risk]. Hence, in a world with taxes, but with no default risks, the *WACC* is an ever-decreasing function of debt, reaching its minimum[11] as debt increases to its theoretical maximum limit of $\tilde{e}_b^*/R_D^*$.[12]

So, the next question is how does the optimal capital structure occur at a finite leverage? To answer this, we need to examine more closely Equation 10, which relates the expected, unlevered earnings [numerator] and the firm's value [denominator] with the *WACC*.

A brief inspection of this equation suggests that, with the numerator held constant and independent of leverage, as required by M&M, the *WACC* would acquire a minimum at some finite leverage if and only if the firm's value, $E + D$, passes through a maximum. The following sections address the above in more detail, as we derive a new model and compare it with the conventional one.

3.1 The "Maximium-value" Approach

The process behind generating the *WACC* curve shall be termed the "maximum-value'' approach because it focuses on obtaining first the curve for the *FV*, without any reference to the *WACC*. Here, a maximum in the *FV* should occur owing to gains from the tax shield on debt being offset by the costs of financial distress (Ross *et al*, 1998).

Computation of the *WACC* follows then naturally by dividing the "expected'' unlevered earnings[13], which should remain constant with leverage, by the *FV*. Thus, with the expected *EBIT* constant, we obtain a minimum in the *WACC* precisely where the *FV* is maximised. This is discussed next in more detail.

Let us refer once again to our original example. The firm has a debt of 80, an equity of 52 and is subjected to a rate of tax of 40% and interest equal to the risk-free rate plus a spread, the latter taking into account the default risk. Let us also, for sake of simplicity, assume that this spread is currently 1.19%, thus yielding an effective, pre-tax cost of debt of 6.19%.

The question now is what would the firm's value be if it operated in a world with no default risk, given its current equity, debt and cost of debt?[14] It is not difficult to argue here that the cost of debt would be exactly the risk-free rate, which subsequently raises the value of debt from the original $D = 80$ to $(80 \times 6.19\%)/5.00\%$, or approximately $D^* = 99$. Therefore, with equity remaining at 52, the value of the company in this idealised, default-free scenario should be 151 instead of the original 132.[15] Also, the idealised leverage, D^*/E, would be 1.90 *versus* the 1.54 that we had earlier for D/E.

Let us now generalise the above and derive the process that leads to the *FV* curve. The process itself is illustrated in Table 3, as well as described in the notes below it.

(1) Beginning at $D = 0$, increase D incrementally and, following the approach outlined in Figure 4, obtain the corresponding ratings, spreads and costs of debt, R_D, at each level of debt.[16] (2) With the cost of debt known at each stage, express the interest paid as the product $R_D D$. Dividing this by the risk-free rate, R_f, therefore, gives the value of the idealised or "virtual'' riskless debt, D^*, at each stage as

$$D^* = \frac{R_D D}{R_f} \tag{12}$$

With R_D greater than R_f, it becomes clear that the value of debt and, hence, the firm falls as the cost of debt rises.

(2) It thus follows that, in this default-free scenario, the value of the unlevered firm, V_u^*, should be

$$V_u^* = E + (1 - T)D^* \tag{13}$$

based on Equation 2. In accordance with M&M's principles, V_u^* should remain constant, independent of D^*. (3) Thus, with V_u^*, as well as T, given, the value of equity, E, and, hence, the firm, $E + D$, could be assessed at each level of D. Figure 5a compares the ideal and the real values and provides proof that, in contrast to its idealised counterpart, $E + D^*$, which lacks a maximum, $E + D$ does indeed possess one. The discrepancy between the two curves is attributable solely to the difference between D^* and D.

We are now also in a position to de-lever the original beta and re-lever it back to compute the corresponding cost of equity and the expected, unlevered earnings at each stage of D. It is important, however, to stress that this de-levering and re-levering of the beta must be conducted using the ratio D^*/E, and not D/E. The rationale for this goes back to the derivation of the levered beta in Equation 9, which was based on the default-free scenario. Therefore, since in this case the current D^*/E is 1.90 and the tax rate 40%, the unlevered beta should be $1.25/[1 + 1.90(1 - 0.4)]$, which equals 0.58. It follows from this that the expected, unlevered earnings, i.e. $\tilde{e}_b(1 - T)$, could be calculated from

$$\tilde{e}_b(1 - T) = [R_f + \beta_L R_p]E + R_D D(1 - T) \tag{14}$$

where R_p is the market risk premium[17], thus yielding a constant of 9.47 throughout[18]. Finally, dividing this by the firm's value, $E + D$, leads to the *WACC* curve shown in Figure 5b. Evidently, because of the constant *EBIT* [see Column 11 in Table 3] the minimum in the *WACC* occurs precisely where the firm's value is maximised.

We end this section with another note, namely, the apparent mismatch between the incremental debt raised and the equity purchased. To illustrate, let us refer again to Table 3, to where the total debt [Column 1] and equity [Column 4] are 120.00 and 16.27, respectively. Our calculations show that raising the debt from 120.00 to 126.67 reduces the equity from the original 16.27 to 5.97. The question, therefore, is how does an added, incremental debt of 6.67 enable an equity purchase of 10.30? The answer is that the balance is funded by the sale of assets, which explains the subsequent decline in the total value of the firm. Another question is what if, instead, one decided to purchase an amount of 6.67 in equity, in an attempt to exactly match the incremental debt raised? This, of course, could be done, but the resulting point will lie on a different *WACC* or *FV* curve.

TABLE 3: A DEMONSTRATION OF THE PROCESS THAT LEADS TO THE FIRM'S VALUE AND, HENCE, THE *WACC* CURVE. THE CURVES THEMSELVES ARE DISPLAYED IN FIGURES 5A-B. THE CURRENT STANDING OF THE FIRM IS HIGHLIGHTED, AND THE OPTIMAL CAPITAL STRUCTURE, WHICH OCCURS AT THE MAXIMUM VALUE AND MINIMUM *WACC*, IS OBSERVED HERE AT A DEBT OF 113 AND A RATING OF BBB. THE PROCESS ITSELF, WHICH TAKES US FROM *D* TO *WACC* IS EXPLAINED IN THE NOTES BELOW THE TABLE.

D (1)	Rating (2)	spread (2)	Interest rate (2)	D* (3)	E (4)	V (5)	V* (6)	D*/E (7)	D/E (8)	beta (based on D*/E) (9)	COE based on beta (10)	Exp. EBIT based on beta (11)	WACC (12)	
0.00	AAA	0.20%	5.20%	0.00	111.42	111.42	111.42	0.00	0.00	0.58	8.50%	9.47	8.50%	
6.67	AAA	0.20%	5.20%	6.93	107.26	113.93	114.20	0.06	0.06	0.61	8.64%	9.47	8.31%	
13.33	AAA	0.20%	5.20%	13.87	103.10	116.44	116.97	0.13	0.13	0.63	8.78%	9.47	8.13%	
20.00	AAA	0.20%	5.20%	20.80	98.94	118.94	119.74	0.21	0.20	0.66	8.94%	9.47	7.96%	
26.67	AAA	0.20%	5.20%	27.73	94.78	121.45	122.52	0.29	0.28	0.69	9.11%	9.47	7.80%	
33.33	AAA	0.20%	5.20%	34.67	90.62	123.96	125.29	0.38	0.37	0.72	9.30%	9.47	7.64%	
40.00	AAA-	0.26%	5.26%	42.07	86.18	126.18	128.25	0.49	0.46	0.75	9.53%	9.47	7.51%	
46.67	AA+	0.44%	5.44%	50.82	80.93	127.60	131.75	0.63	0.58	0.80	9.82%	9.47	7.42%	
53.33	AA-	0.64%	5.64%	60.20	75.30	128.64	135.50	0.80	0.71	0.86	10.18%	9.47	7.36%	
60.00	A+	0.82%	5.82%	69.83	69.52	129.52	139.35	1.00	0.86	0.93	10.61%	9.47	7.31%	
66.67	A	0.97%	5.97%	79.55	63.69	130.36	143.24	1.25	1.05	1.02	11.12%	9.47	7.27%	
73.33	A	1.09%	6.09%	89.30	57.84	131.18	147.14	1.54	1.27	1.12	11.74%	9.47	7.22%	
80.00	**A-**	**1.19%**	**6.19%**	**99.04**	**52.00**	**132.00**	**151.04**	**1.90**	**1.54**	**1.25**	**12.50%**	**9.47**	**7.18%**	**Current**
86.67	A-	1.27%	6.27%	108.77	46.16	132.83	154.93	2.36	1.88	1.41	13.45%	9.47	7.13%	
93.33	A-	1.35%	6.35%	118.48	40.33	133.67	158.81	2.94	2.31	1.61	14.67%	9.47	7.09%	
100.00	A-	1.41%	6.41%	128.18	34.51	134.51	162.70	3.71	2.90	1.88	16.30%	9.47	7.04%	
106.67	BBB+	1.46%	6.46%	137.87	28.70	135.36	166.57	4.80	3.72	2.27	18.59%	9.47	7.00%	
113.33	**BBB**	1.50%	6.50%	147.45	22.95	**136.29**	170.40	6.42	4.94	2.83	21.99%	9.47	**6.95%**	**Optimal**
120.00	BBB-	1.61%	6.61%	158.59	16.27	136.27	174.86	9.75	7.38	4.00	28.98%	9.47	6.95%	
126.67	BB	1.94%	6.94%	175.76	5.97	132.63	181.73	29.46	21.23	10.90	70.38%	9.47	7.14%	

(1) Debt increased incrementally and calculations carried out at every stage of debt.
(2) Rating and the corresponding spread and interest rate or pre-tax cost of debt obtained from the process described in Section 2.3 or Figure 4.
(3) The "virtual" value of debt, calculated using Equation 12.
(4) Value of equity calculated from the relationship involving the unlevered firm's value, i.e. using D in Equation 2.*
(5) Firm's realised value, D + E.
(6) V calculated as E + D* or Equation 3, based on D*. The two should yield the same result.*
(7) The "virtual" leverage, based on D.*
(8) Realised leverage, D/E.
(9) The beta calculated using Equation 9, based on D/E.*
(10) The cost of equity, based on the risk-free rate, risk premium and the just-calculated beta.
(11) The after-tax expected EBIT, calculated using Equation 14.
(12) The WACC, computed via either Equation 4d or 10, using V from Column 5 for the firm's value and EBIT from Column 11.

3.2 The Conventional Approach

The conventional approach, which is used more commonly in practice, is in many ways similar to the above, except that all calculations are carried out using the realised leverage, D/E, instead of the idealised, D^*/E. We will prove here that this approach is flawed.

The process that underlies the conventional approach is described in Table 4 and the notes that follow it. At start, we need the unlevered firm's value. This is obtained from Equation 2. For our specific example, V_u would then be $52 + 80 \times (1 - 40\%)$, or 100.

Very briefly, the process is as follows: (1) we begin with a debt, D, of zero and increase it incrementally. (2) Similar to the discussion related to Table 3, we compute, for each stage, the corresponding rating, spread and the cost of debt. (3) The equity value is subsequently computed at each stage using Equation 2 again, with $V_u = 100$. (4) This then leads to the realised or actual firm's value, $D + E$, which only rises as D increases. (5) The actual leverage, D/E, comes next, which, along with the given tax rate of 40%, yields the levered beta at each stage. The unlevered beta in this case is, however, $(1.25/[1 + 1.54(1 - 0.4)]) = 0.65$, where the 1.54 is the current D/E, i.e. 80/52. (6) With the levered beta known at each stage, the cost of equity, R_E, and, hence, the expected, unlevered earnings could be computed, the former coming from

$$R_E = R_f + \beta_L R_p \tag{15}$$

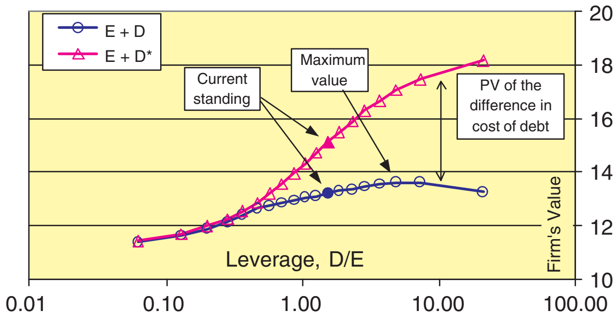


Figure 5a: The firm's value as a function of leverage. The numbers are taken from Table 3, which provides an example of the process behind the "maximum-value" approach. The triangles depict the firm's value in a world with no default risk, and the circles represent the value in a world with default risk. The two solid points depict the firm's current standing, which is also highlighted in Table 3. Note the absence of a maximum in the idealised case (triangles) and its presence in the realistic one (circles)

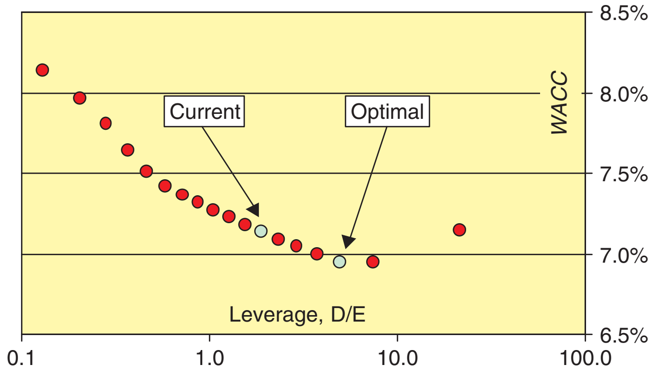


Figure 5b: The *WACC* curve obtained from the "maximum-value" approach, as outlined in Section 3.2. The optimal point, which corresponds to the minimum *WACC*, is highlighted

and the latter from Equation 14. Lastly, (7) the *WACC* is evaluated using Equation 10.

The conventional approach, although regularly used in practice, appears to be flawed for the following 2 reasons. Firstly, we note from Table 4, as well as in Figure 6, that the expected, unlevered earnings, as calculated in this conventional manner using D/E rather than D^*/E, is not constant, but varies with leverage. This is inconsistent with the original contention of M&M that this number must be constant, independent of leverage. Secondly, we note that the firm's value, which is depicted in Column 4, is ever increasing and, hence, does not pass through a maximum, as it should in theory. Moreover, there appears to be a minimum *WACC*, which coincides with a debt, D, of 120. The mere observation that the firm's value does not pass through a maximum while the *WACC* possesses a minimum is, in itself, contradictory.

The root of the above problems lies in applying Equations 9 and 13 to a cost of debt that is variable. Here, we need to recall that derivation of both equations, which, respectively, relate to the beta and the unlevered value of the firm, were based on the presumption that the cost of debt remained constant, not variable.

4 An Effect of the Tax Rate on the Optimal Credit Rating

There is an additional issue that crops up when assessing a company's capital structure. It concerns the impact of the tax rate on the optimal credit rating[19].

It is widely believed in practice that a typical company should have its optimal capital structure coincide with a specific credit rating. Some maintain it should be BBB, others A- and so on. Apparently, there seems to be no common answer to this question. We will prove here that the tax rate affects the optimal credit rating in a way that firms that operate in different tax-rate environments must aim for different credit ratings consistent with their optimal capital structure. Let us demonstrate.

Consider a firm that does not pay any taxes, but is prone to default risk. The fact that this firm can claim no interest-related tax benefits implies that its value curve should maximise at zero leverage. The zero-leverage point would, in turn, be compatible with a AAA rating. Thus, one could argue here that the optimal credit rating for a firm that pays no taxes, but is subject to default risk, is AAA, or somewhere close to it.

We extended this argument here a bit further and generated Table 5, which depicts how the tax rate might affect the optimal rating. This table was produced by applying different tax rates to the spreadsheet behind Table 3. Assuming that the same credit spread shown in Table 2 applies under all tax rates[20], we note that, in general, higher tax rates lead to lower optimal ratings and vice versa. In this particular case, for instance, an optimal BBB occurs at a tax rate of between 33% and 40%, whereas an optimal AA- occurs at a tax rate of 30%. This distribution, however, is expected to be highly dependent on the rating-to-spread relationship but, qualitatively, this general behaviour should hold by and large.

5 Conclusions

We have so far discussed the basic principles that underlie the M&M capital structuring theorems, added in the effects of default risk and derived an analytical technique by which the *FV* and the *WACC* curves could be produced. In the process, we have, hopefully, also shed light on problems that are inherent within the conventional approach, and explained why the alternative, which we have termed here as the "maximum-value" approach, should be implemented instead.

TABLE 4: A DEMONSTRATION OF HOW THE CONVENTIONAL APPROACH MAY LEAD TO A VARIABLE *EBIT*, AS WELL AS TO A MINIMUM WACC IN THE ABSENCE OF A MAXIMUM IN THE FIRM'S VALUE

D (1)	Interest rate (2)	E (3)	V (4)	D/E (5)	beta (based on D/E) (6)	COE based on beta (7)	Exp. EBIT based on beta (8)	WACC based on beta1 (9)	
0.00	5.20%	100.00	100.00	0.00	0.65	8.90%	8.90	8.90%	
6.67	5.20%	96.00	102.67	0.07	0.68	9.06%	8.91	8.68%	
13.33	5.20%	92.00	105.33	0.14	0.71	9.24%	8.92	8.46%	
20.00	5.20%	88.00	108.00	0.23	0.74	9.43%	8.92	8.26%	
26.67	5.20%	84.00	110.67	0.32	0.77	9.64%	8.93	8.07%	
33.33	5.20%	80.00	113.33	0.42	0.81	9.88%	8.94	7.89%	
40.00	5.26%	76.00	116.00	0.53	0.86	10.13%	8.96	7.73%	
46.67	5.44%	72.00	118.67	0.65	0.90	10.42%	9.02	7.60%	
53.33	5.64%	68.00	121.33	0.78	0.96	10.74%	9.11	7.50%	
60.00	5.82%	64.00	124.00	0.94	1.02	11.09%	9.19	7.42%	
66.67	5.97%	60.00	126.67	1.11	1.08	11.50%	9.29	7.33%	
73.33	6.09%	56.00	129.33	1.31	1.16	11.96%	9.38	7.25%	
80.00	**6.19%**	**52.00**	**132.00**	**1.54**	**1.25**	**12.50%**	**9.47**	**7.18%**	**Current**
86.67	6.27%	48.00	134.67	1.81	1.35	13.13%	9.56	7.10%	
93.33	6.35%	44.00	137.33	2.12	1.48	13.86%	9.65	7.03%	
100.00	6.41%	40.00	140.00	2.50	1.63	14.75%	9.75	6.96%	
106.67	6.46%	36.00	142.67	2.96	1.81	15.83%	9.84	6.89%	
113.33	6.50%	32.00	145.33	3.54	2.03	17.19%	9.92	6.83%	
120.00	6.61%	28.00	148.00	4.29	2.32	18.93%	10.06	**6.80%**	**Min. WACC**
126.67	6.94%	24.00	150.67	5.28	2.71	21.25%	10.37	6.88%	

(1) Debt increased incrementally and calculations carried out at every stage of debt.
(2) Interest rate or pre-tax cost of debt calculated based on the process described in Section 2.3 or Figure 4.
(3) Value of equity calculated from the relationship involving the unlevered firm's value, i.e. Equation 2.
(4) The firm's value, V, calculated as E + D, using Columns 1 & 3, or Equation 3, based on D.
(5) The leverage, based on D & E.
(6) The beta calculated using Equation 9, based on D/E.
(7) The cost of equity, based on the risk-free rate, risk premium and the just-calculated beta.
(8) The after-tax expected EBIT, calculated using Equation 14.
(9) The WACC computed via 10, using D for the debt.

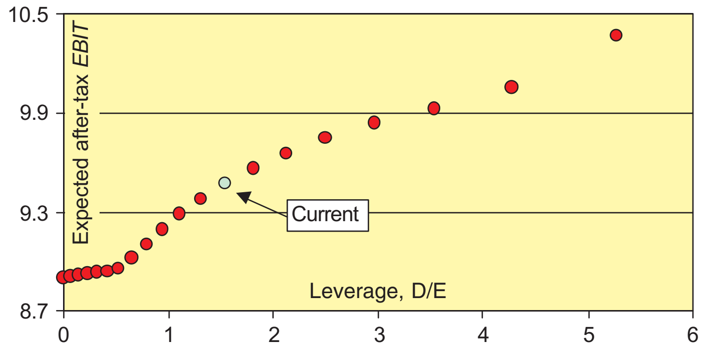


Figure 6: The expected, unlevered earnings plotted against leverage. The calculations are based on Equation 14, using the data in Table 4. This contradicts one of M&M's important principles, namely that the *EBIT* remains constant, independent of leverage

Basically, one of the issues surrounding the conventional approach surface because it conflicts with one of M&M's original hypotheses – namely that the expected, unlevered earnings should remain constant, independent of leverage. Another issue pertains to the lack of a maximum in the firm's value. Yet, putting the two together leads to a minimum in the WACC curve, which, obviously, is in contradiction with the absence of a maximum in the firm's value. Never the less, this problem is resolved by incorporating the so-called "maximum-value'' approach.

Without having to rely on the *EBIT*, the maximum-value approach seeks the leverage at which the *FV* is maximised. This method begins by converting the realised debt, *D*, into its idealised counterpart, D^*, via Equation 12. The difference between D^* and *D* is the present value of the additional interest payments arising from the default risk. Once obtained, computation of the equity value, *FV*, beta, etc. follows naturally. It is important to note here that the de-levering and re-levering of the firm's beta must be carried out relative to the idealised debt-to-equity ratio, i.e. D^*/E, rather than the actual, D/E. With this, the calculated

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TABLE 5 EFFECT OF TAX RATE ON THE RATING AT THE OPTIMAL CAPITAL STRUCTURE. OBSERVE THAT AT A ZERO TAX RATE, THE OPTIMAL CAPITAL STRUCTURE OCCURS AT *D* = 0, WHICH, OF COURSE, COINCIDES WITH A RATING OF *AAA*. ALSO, NOTE THAT AS THE TAX RATE INCREASES, THE OPTIMAL RATING DROPS. THE RELATIONSHIP BETWEEN THE TWO, WHICH IS NOT NECESSARILY LINEAR, DEPENDS STRONGLY ON THE UNDERLYING CREDIT SPREAD AND CREDIT-RATING MODEL.

Tax rate = 0%		Tax rate = 10%		Tax rate = 20%		Tax rate = 30%		Tax rate = 31%		Tax rate = 33%		Tax rate = 40%	
Value, E + D	**Rating**	**Value, E + D**	**Rating**	**Value, E + D**	**Rating**	**Value, E + D**	**Rating**	**Value, E + D**	**Rating**	**Value, E + D**	**Rating**	**Value**	**Rating**
151.04	**AAA**	141.13	AAA	131.23	AAA	121.33	AAA	120.34	AAA	118.35	AAA	111.42	AAA
150.77	AAA	141.56	AAA	132.35	AAA	123.14	AAA	122.22	AAA	120.38	AAA	113.93	AAA
150.50	AAA	141.99	AAA	133.47	AAA	124.95	AAA	124.10	AAA	122.4	AAA	116.44	AAA
150.24	AAA	142.41	AAA	134.59	AAA	126.77	AAA	125.98	AAA	124.42	AAA	118.94	AAA
149.97	AAA	142.84	AAA	135.71	AAA	128.58	AAA	127.87	AAA	126.44	AAA	121.45	AAA
149.70	AAA	**143.27**	**AAA**	136.83	AAA	130.39	AAA	129.75	AAA	128.46	AAA	123.96	AAA
148.96	AAA-	143.27	AAA-	**137.57**	**AAA-**	131.87	AAA-	131.30	AAA-	130.16	AAA-	126.18	AAA-
146.89	AA+	142.06	AA+	137.24	AA+	132.42	AA+	131.94	AA+	130.97	AA+	127.60	AA+
144.17	AA-	140.29	AA-	136.40	AA-	**132.52**	**AA-**	132.13	AA-	131.36	AA-	128.64	AA-
141.20	A+	138.28	A+	135.36	A+	132.44	A+	**132.15**	**A+**	131.57	A+	129.52	A+
138.15	A	136.20	A	134.25	A	132.30	A	132.11	A	131.72	A	130.36	A
135.07	A	134.10	A	133.13	A	132.15	A	132.05	A	131.86	A	131.18	A
132.00	**A-**	**132.00**	**A-**	**132.00**	**A-**	**132.00**	**A-**	**132.00**	**A-**	**132**	**A-**	**132.00**	**A-**
128.94	A-	129.91	A-	130.88	A-	131.86	A-	131.95	A-	132.15	A-	132.83	A-
125.89	A-	127.83	A-	129.78	A-	131.72	A-	131.92	A-	132.31	A-	133.67	A-
122.85	A-	125.77	A-	128.68	A-	131.60	A-	131.89	A-	132.47	A-	134.51	A-
119.83	BBB+	123.71	BBB+	127.60	BBB+	131.48	BBB+	131.87	BBB+	132.65	BBB+	135.36	BBB+
116.92	BBB	121.76	BBB	126.61	BBB	131.45	BBB	131.93	BBB	**132.9**	**BBB**	**136.29**	**BBB**
				124.35	BBB-	130.31	BBB-	130.91	BBB-	132.1	BBB-	136.27	BBB-
										127.26	BB	132.63	BB

🟩 **Current** 🟧 **Optimal**

expected *EBIT* remains constant and independent of leverage and, hence, becomes consistent with M&M's requirement. The result, therefore, is a unique optimal capital structure, regardless of whether it falls out from the *FV* or the *WACC* curve.

Moreover, we also came across an effect that the tax rate might have on the optimal credit rating. We found, in this instance, that in high tax-rate environments, firms may be able to achieve their optimal capital structure at a lower credit rating. In low tax-rate environments, however, they may not be so lucky. The reason for this is that taxes can create benefits in ways other than increasing the *FV* or reducing the *WACC*. These other benefits could include, for instance, an added flexibility for the firm to aim for a lower rating and, still, be able to optimise its capital structure.

Overall, it should be stressed that the numerical outcomes presented in this paper are sensitive to the spread-ratings relationship used [Table 2] and, hence, any other relationship would certainly generate different results. Qualitatively, however, the general trends are expected to remain the same, regardless of this functional relation.[21]

FOOTNOTES & REFERENCES

1. Those interested in the second issue may refer to, among many others, Mao (1979), Rege and Baxter, (1982), Paulo (1992), Wang (1994) and Mills (2001).

2. This constitutes a cost to the firm, but a return to the equity investor.

3. Courses on optimising the capital structure form an integral part of the training in almost all major academic and financial institutions.

4. The capital, i.e. $E + D$, has generally, although mistakenly, been used as the constant that relates debt to equity.

5. The "expected" *EBIT*, as described in the preceding paper, is that which reconciles the "expected" return in the income statement with the stock's total rate of return. The latter is also known as the cost of equity.

6. The distinction between the two is discussed in Brealey and Myers (1996). In practice, however, one must implement the market value of equity when calculating the firm's value.

7. The very notion of beta is littered with controversies. While some works strongly support it, others advocate its "death " [Grinold (1993) and references therein]. Moreover, some say that it should be industry based, while others argue that it should be the firm's.

We shall avoid delving into these issues here and will, instead, utilise the beta as objectively as possible by taking it as a "given."

8. Note that this emanates from the CAPM.

9. The spread on the firm's cost of debt over the risk-free rate would be identically zero in this case.

10. It is important to emphasise that any impact that bankruptcy costs might have on the spreads and, hence, the cost of debt, has been excluded from this paper. The reason for this is that such costs tend to be more subjective and difficult to predict than the probability of default. Adding on such costs will only make this work more quantitative and, perhaps, even more difficult to read.

11. The point of minimum *WACC* is the optimal capital structure since it signifies the minimum cost of capital needed to operate the company.

12. This is obtained by equating Equation 4b to zero and solving for *D*.

13. The "expected" unlevered earnings may, in practice, be calculated using CAPM, as discussed later, or taken as either a historical average or an equivalent to the analysts' consensus forecast.

14. Refer to the "idealised" scenario discussed earlier in Section 2.1.

15. The difference of 19 is, in fact, related to the present value of the losses incurred by paying the additional interest over the risk-free rate.

16. For $D = 0$, the rating and the corresponding spread would very likely be AAA and *0.20%*, respectively [see Table 2].

17. The market risk premium is taken to be 6% here.

18. As mentioned in an earlier footnote, it might be also appropriate to use instead a historical average or a consensus forecast of the *EBIT*.

19. By "optimal credit rating" we mean the credit rating that corresponds to the optimal capital structure.

20. We have not yet tested, or checked in the literature, as to whether or not this is a valid assumption.

21. That is, as long as the spread increases with deteriorating credit rating.

- Brealey, R.A. and Myers, S.C., Principles of Corporate Finance, 5th Ed., McGraw-Hill, 1996.
- Chew, D.H., Jr. (Ed.) The New Corporate Finance, 2nd Ed., Irwin McGraw-Hill, 1998.
- Cohen, R.D., "An Implication of the Modigliani-Miller Capital Structuring Theorems on the Relation Between Equity and Debt," *Citibank internal memo*, May 2001.
- Damodaran, A., Lecture notes on capital structure, September 2000.

Grinold, R.C., "Is Beta Dead Again?" *Financial Analysts Journal*, pp 28-34 (July-August 1993).

- Mao, J.C.T., "The *WACC* as an Investment Hurdle Rate," *Cost and Management 53*, p. 28 (1979).
- Mills, R.W., "Developments in Estimating the Cost of Capital," *Manager Update 2*, pp. 35-46 (2001).
- Modigliani, F., and Miller, M.H., "The Cost of Capital, Corporation Finance and the Theory of Investment," *American Economic Review 48*, pp. 261-297 (1958).
- Paulo, S. "The Weighted Average Cost of Capital: A Caveat," *The Engineering Economist 37*, pp. 178-183 (1992).
- Rege, U.P. and Baxter, G.C., "Weighted Average Cost of Capital: A Tool for Decision Making," *Cost and Management 56*, p. 36 (1982).
- Ross, S.A., R.W. Westerfield and B.D. Jordan, Fundamentals of Corporate Finance, 4th Ed., McGraw-Hill, 1998.
- Wang, L.K., "The Weighted Average Cost of Capital and Sequential Marginal Costing: A Clarification," *The Engineering Economist 39*, p. 187 (Winter 1994).