The Optimal Capital Structure of Depository Institutions[1,2]

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Abstract
We derive here a fundamental model for the capital structure of depository institutions. The derivation centres on the basic Modigliani-Miller methodology, but instead of using a constant EBIT, as classically done for corporate firms, it implements a variable one, which hinges on the interest earnings from the asset-based loans made to the borrower. Following this, the effect of risk and credit spreads of both, the lender and borrower, are introduced and the impact of leverage on certain basic ratios, in particular the return on equity, is assessed.

The outcome of this work is twofold. Firstly, it highlights some of the main differences that exist between the treatment of the capital structure of corporate firms and depository institutions. And, secondly, it demonstrates that the optimal capital structure of a depository institution is not as easily identifiable as that of a corporate's. The reasons for this include, among others, (i) the existence of regulatory capital restrictions, (ii) an inter-dependence between the borrower and the lender and (iii) a dramatic change in the behaviour of the return on equity with respect to leverage when risks and credit spreads of both, lender and borrower, are accounted for.

1 Introduction

The impact of regulations on depository institutions[4] [hereafter also referred to as lending institutions, lenders or banks] has turned capital structuring into an important area of concern and interest. Here as well, as in the case of corporate firms, the attention revolves around trying to identify the optimal capital structure, as this, presumably, enables the organisation to operate more efficiently. Unlike corporate firms, however, where the Modigliani-Miller [M&M] theorems have clear-cut consequences, applying capital structuring to banks is more subjective. The reason for this is that here capital structuring relies heavily on risk management and value creation, two factors that are tightly entwined, owing to the nature of the business [*Schroeck* (2002) and *Fabozzi* (1999), among many others].

There is also no need to mention that the amount of literature covering this area is considerable and, thus, any effort to undermine even a fraction of it would take the attention away from our objectives. Never the less, it would be helpful to address some of the relevant, but important, issues as we go along.

One should recognise that, in contrast to a corporate firm, there is an underlying problem in assessing the capital structure of a bank. This problem rests mainly on the lack of clarity on how one could define, let alone determine, the location of the optimal. The reason for this is rooted in the differences in the ways the two types of organisations operate. For instance, while a corporate firm generates income from rendering services and/or selling manufactured products, a simple bank brings in revenues by *lending* its assets. To further complicate things, the type of borrower also plays a vital role, particularly when regulatory capital constraints are enforced on the lender.

It is, therefore, not hard to imagine that for a variety of reasons—namely (1) the fundamental discrepancies between how banks and corporate firms operate and (2) with risk and value management come complex interactions between the lender and borrower (*Mason*, 1995)—the task of determining the optimal capital structure of a bank, as opposed to that of a corporate firm, is far from trivial. With this in mind, we intend here to add some insight into this process, albeit in a simplistic manner, as we try to (1) describe the mechanics that intertwine a lender and a borrower and (2) establish a logical framework for defining the optimal capital structure of a lending institution.

Our approach to the above will be as follows. We start with an overview of the application of the M&M ideology to a corporate, hoping to shed light on some of the key elements that differentiate between such an organisation and a lending institution. In due course, we demonstrate how these features emanate from the differences in the "fundamental constants,'' as well as the levered and unlevered betas, which typify the two types of establishments. Next, we explain the basic role of regulatory capital, as this tends to play a significant part in the operation of banks. We then go on to discuss the impact of the risk of default on both, the lender and the borrower, which, subsequently, leads to four unique scenarios. These scenarios should, hopefully, help elucidate the notion of an optimal capital structure for depository institutions. And last, but not least, we describe how one could implement such a concept in practice.

2 Application of the M&M Methodology to a Corporate Firm

Although the basic M&M theory for corporate firms is well known and presents itself in almost every text that covers the fundamentals of corporate finance [see, for instance, *Brealey and Myers* (1996) or *Ross et al* (1998)], we have decided to re-derive it here since much of the work that lies ahead will, in one way or another, be related to it. For convenience, our derivation at this stage is kept rudimentary, based on the assumption of no default risk, and will be illustrated step by step via the simplified financial statement displayed in Figure 1.[5]

Income Statement	
EBIT	20
Interest (at 5%)	-5
EBT	15
Tax (at 40%)	-6
Net Profit	9
Balance Sheet	
Total Assets	150
Debt	100
Equity	50
Debt + Equity	150
Ratios	
Leverage, D/E	2
ROE, Net Profit/Equity	18%

Figure 1: A simplified financial statement, where the balance sheet is limited to debt and equity. We have assumed a cost of debt of 5% and a tax rate of 40%

Prior to plunging into the equations, it might be worthwhile to add a few words on the fundamentals that underlie M&M's capital structuring. These fundamentals consist of three propositions, dealing with the (i) impact of tax on firm's value, (ii) effect of leverage on the return on equity and (iii) irrelevance of dividends on shareholder value. The first two of these, which are most pertinent to this work, will, essentially, fall out as we progress with the derivation.

In reference to Figure 1, which relates to a corporate firm, the *EBIT*[6], e_b, may be expressed in terms of the return on equity, R_E, and cost of "risk-less" debt[7], R_D^*,as

$$e_b(1-T) = R_E E + R_D^* D^*(1-T) \tag{2.1}$$

where E, D^* and T, respectively, are the equity, risk-less debt and tax rate. In this case, therefore, R_E is equal to the net profit, 9, divided by the equity, 50, resulting in 18%. An important feature that sets apart a typical corporate firm from a lending institution is that the *EBIT* of the former, which is an operating income, remains theoretically constant, independent of T, E and D^*.

Next, on defining R_u as the cost of operating the unlevered firm, which should stay invariable, and V_u as the value of the unlevered firm, we obtain:[8]

$$V_u = \frac{e_b(1-T)}{R_u} \equiv \frac{R_E E}{R_E} + \frac{R_D^* D^*(1-T)}{R_D^*} = E + D^*(1-T) \tag{2.2}$$

after imposing the M&M ideology on V_u; e.g. V_u is the sum of the present values of the net profit, $R_E E$, and the after-tax interest payment, $R_D^* D^*(1-T)$.[9] Therefore, with e_b and R_u constant, the unlevered value of the firm, V_u, remains constant as well, and so does the quantity $E + D^*(1-T)$. In fact, the quantity $E + D^*(1-T)$ represents the fundamental constant from which the levered value of the firm may be computed.[10]

Having established the above, one could now argue that the difference between the levered value of the firm, V_L, which is simply $E + D^*$, and its unlevered counterpart, V_u, denoted by Equation 2.2, is the value added to the firm by the interest tax shield. This amounts to the product D^*T, which works out to be 40 in the case of Figure 1. It, thus, follows that as debt increases, the value of the corporate firm increases as well, albeit in the linear fashion illustrated in Figure 2, with the slope of the line equal to the tax rate, T. This, effectively, proves M&M's first proposition, concerning the impact of tax on firm's value.

An extension of the above leads to the effect of leverage on the return on equity, R_E. Going back to Equations 2.1 and 2.2 and combining, we obtain

$$R_E = R_u + (R_u - R_D^*)(1-T)\phi^* \tag{2.3}$$

where ϕ^* is the [risk-less] leverage defined as

$$\phi^* \equiv \frac{D^*}{E} \tag{2.4}$$

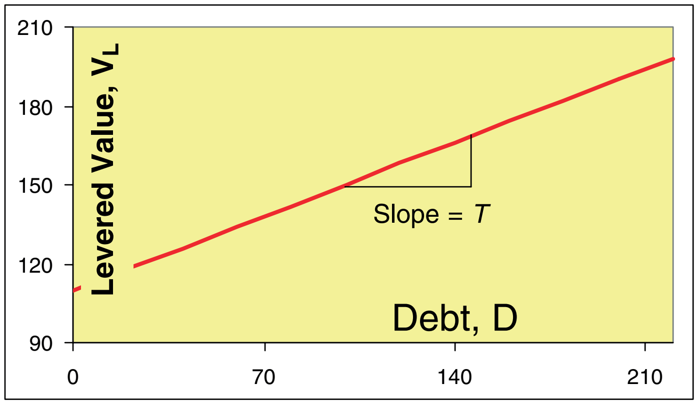

Figure 2: The impact of debt, *D*, on the levered value of the corporate firm. The slope of the curve is the tax rate, *T*, and the intercept is the unlevered value of the firm, which is 110, based on Equation 2.2 and using the financial statement in Figure 1

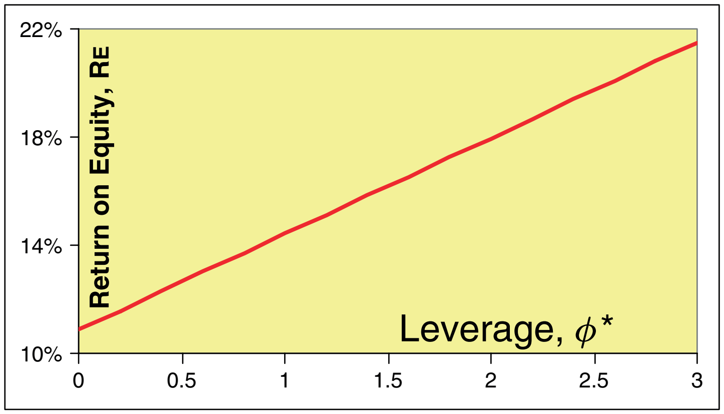

Figure 3: An example of the return on equity, R_E, *vs* the risk-less leverasge, ϕ^*, for a corporate firm. This is based on Equation 2.3

Based on Figure 1, therefore, with R_u equal to 10.9% [see Footnote 9] and R_D^* and T equal to 5% and 40%, respectively, we arrive at Figure 3, which displays the behaviour of R_E as a function of ϕ^*. The relationship is again linear, as in Figure 2, stemming from the assumption that the firm is free from the risk of default. This, essentially, constitutes M&M's second proposition, relating the return on equity to tax and leverage. With the above in mind, we are now in a position to apply the same technique to a depository institution and, hence, derive relationships analogous to Equations 2.1–2.3.

3 Application to a Depository Institution

We have so far demonstrated the famous M&M's treatment of a corporate's capital structure. The important feature here is the "fundamental constant," which, in this case, turns out to be the firm's unlevered value, V_u, represented by Equation 2.2. In practice, this is used to extract the value of the firm as it varies its level of debt or leverage.

It should be emphasised once more that derivation of 2.2 involves an environment that is free from the risk of default. This is portrayed by a cost of debt that remains constant, equal to R_D^*, and independent of leverage. The asterisk on R_D signifies this restriction. Thus, removing this constraint and incorporating the effects of default risk and credit spread should, not surprisingly, complicate the methodology for determining V_u, although it still remains tractable [see, for example, *Cohen* (2000b) for the approach].

In the sections that follow, we utilise a similar approach to develop the fundamental constant that, this time, relates instead to a depository institution. Ideally, such an establishment would obtain funds at low rates from deposits, as well as from government and/or interbank loans, and lend them, together with its own equity, to another firm at a margin[11]. This margin turns out to be the main source of profit generation for such an institution.

3.1 The use of the M&M principle to derive the fundamental constant for a depository institution

An approach similar to the above shall now be taken to derive the fundamental constant for an institution that earns its revenues from lending its assets. For illustration, we will, once again, refer to a basic financial statement. Simplification is needed here in order to shed light on the fundamentals that underlie capital structuring and its optimisation. Without properly understanding these, it is practically useless to chase after more complicated cases.

Let us now begin with the usual assumption that the rates of lending and borrowing are independent of leverage, as it is typically done under the classical M&M approach. We denote these rates [or costs] by R_T^* and R_D^*, respectively, the former being the rate that the lender charges the borrower and the latter the rate the lender is charged to service its own debt[12]. More realistically, R_T and R_D would vary with certain parameters, as the risk of default and credit rating get affected. We shall touch on this shortly.

In view of the above, we re-apply Equation 2.1 to the lender and obtain:

$$R_T^*(E + D^*)(1 - T) = R_E E + R_D^* D^*(1 - T) \tag{3.1}$$

where, this time, we have equated the operating income, e_b, to the *net* revenues[13] generated from lending the assets, $E + D^*$, at the rate of R_T^*, i.e.

$$e_b = R_T^*(E + D^*) \tag{3.2}$$

A simple illustration of this, depicting the effects of a change in debt and equity on leverage and return on equity, is displayed in Figures 4a–b. As mentioned earlier, Equation 3.2 constitutes one of the crucial elements that distinguish between how corporate firms and banks derive their revenues.

Income Statement	
Operating Income (at 5% of assets)	7.5
Interest (at 4%)	-4
EBT	3.5
Tax (at 40%)	-1.4
Net Profit	2.1
Balance Sheet	
Total Assets	150
Debt	100
Equity	50
Debt + Equity	150
Ratios	
Leverage, D/E	2
ROE, Net Profit/Equity	4%

(a)

Income Statement	
Operating Income (at 5% of assets)	10
Interest paid (at 4%)	-6
EBT	4
Tax (at 40%)	-1.6
Net Profit	2.4
Balance Sheet	
Total Assets	200
Debt	150
Equity	50
Debt + Equity	200
Ratios	
Leverage, D/E	3
ROE, Net Profit/Equity	5%

(b)

Figure 4: A simplistic financial statement for a lending institution. Here, the operating income comes from lending assets [$D+E$] at some rate, R_T, while interest is paid on debt, D, at rate R_D—in this case 5% and 4%, respectively. In Figure 4a, the debt is 100, whereas in Figure 4b, it is 150. The dependence of the operating income on asset size and/or leverage is what distinguishes the above from a corporate's financial statement, where, in theory, the latter's operating income [*EBIT*] remains constant, independent of leverage

Next, in order to get the fundamental constant characterising a depository institution, we follow exactly the same procedure as before in going from Equations 2.1 to 2.2—that is, introduce a constant discount rate, α,such that it satisfies the following relationship:

$$\begin{aligned}\frac{R_T^*(E+D^*)(1-T)}{\alpha} &\equiv \frac{R_E E}{R_E} + \frac{R_D^* D^*(1-T)}{R_D^*} \\ &= E + D^*(1-T)\end{aligned} \tag{3.3}$$

where the left-hand side is set equal to the individual present values of $R_E E$ and $R_D^* D^*(1-T)$—i.e. E and $D^*(1-T)$—respectively. On dividing both sides of 3.3 by $E+D^*$ and implementing 2.4, we obtain:

$$\frac{R_T^*(1-T)}{\alpha} = \frac{1+\phi^*(1-T)}{1+\phi^*} \tag{3.4}$$

With each of the parameters on the left-hand side of 3.4,[14] namely, R_T^*, T and α, held constant, it should, therefore, only be the case that

$$\phi^* = \text{constant} \tag{3.5}$$

which, hence, constitutes the fundamental constant relating to a depository institution. Consequently, just as a corporate firm should follow the path $E+D^*(1-T)$ as its characteristic constant, a lending establishment should instead move along $\phi^* \equiv D^*/E = \text{constant}$t.[15] To demonstrate, if the lender's financial statement were represented by Figure 4b, which has a leverage of 3, then an increase of 300 in debt, taking it to 450, would necessitate an additional equity issuance of 100 if the lender were to maintain the same leverage ratio, ϕ^*, of 3. Thus, as the bank progresses along its characteristic constant in the process of raising its debt from 150 to 450, its overall value should rise by 400, owing to the additional 100 in new equity it has to issue. This contrasts sharply to a corporate's capital structure, where, according to M&M, a firm that proceeds along its characteristic constant as it changes its capital structure, is able to *buy back* a portion of its equity as it takes on additional debt (*Cohen*, 2001a).

Let us now generalise the above. Since the value of the levered bank, V_L, is $E+D^*$, we obtain

$$V_L = \left(1+\frac{1}{\phi^*}\right)D^* \tag{3.6}$$

upon utilising 2.4. This, in association with 3.5, implies that the value of a risk-less lender increases linearly as it takes on more debt, just as a corporate's levered value does, but here it happens at a rate of $(1+1/\phi^*)$ as opposed to the tax rate, T. A consequence of this is that the interest-tax shield is not an effective way for adding value when lending assets is the sole source of revenue generation.

The above enables us now to develop the relationship between the return on equity, R_E, and leverage, ϕ^*, for a lending institution. We revert to 3.1, which, in association with 2.4, yield

$$R_E = R_T^*(1-T) + (R_T^* - R_D^*)(1-T)\phi^* \tag{3.7}$$

after re-arrangement. This resembles Equation 2.3 in that both are linear in ϕ^*. The main difference, notwithstanding, is the presence of the lending rate, R_T^*, instead of R_u, the latter applying to a corporate firm.

In light of the above, we try next evaluating the impacts of risks and credit spreads of both, lender and borrower, on the return on equity. However, it would be more appropriate to derive first the relationship between the levered and unlevered betas, as it relates to a depository institution, and compare it with its corporate counterpart.

3.2 The relationship between the levered and unlevered betas of a depository institution

Just as a corporate firm's beta varies with leverage, a lending institution's should do so as well. Never the less, the disparities between the two betas, the lender's and the corporate firm's, arise predominantly from the differences between the fundamental constants. Let us illustrate.

In terms of the return on equity, R_E, the risk-free rate, which is assumed here to be equal to R_D^*, and the market's risk premium, R_P, the levered beta, β_L, is generally defined as:

$$\beta_L \equiv \frac{R_E - R_D^*}{R_P} \tag{3.8a}$$

and its unlevered counterpart, β_u, as

$$\beta_u \equiv \frac{R_E(\phi^*=0) - R_D^*}{R_P} \tag{3.8b}$$

where $R_E(\phi^*=0)$ denotes the return on equity evaluated at zero leverage, i.e. letting $\phi^*=0$ in Equation 3.7. We could now obtain the relationship between β_L and β_u for a depository institution by eliminating R_P from 3.8a and 3.8b and substituting Equation 3.7 for R_E. The result is:

$$\beta_L = \beta_u \left[1 + \left(\frac{R_T^* - R_D^*}{R_T^*(1-T) - R_D^*} \right) \phi^*(1-T) \right] \tag{3.9}$$

which resembles that of a corporate firm's[16], except for the appearance of an extra term containing the margin between R_T^* and R_D^*. It is interesting to note here that unlike for a corporate firm, where the levered beta is always an increasing function of leverage, a depository institution's levered beta could very well decrease with leverage, especially if the tax rate, T, is high enough to render $R_T^*(1-T) - R_D^* < 0$.[17]

Our discussions so far have centred on risk-less cases, where, essentially, the rates of lending and borrowing are held constant. Evidently, this is far from reality, as every organisation, be it corporate or financial, is vulnerable to defaulting on its debts. It, therefore, becomes necessary to generalise our analysis by including the effects of default as well. Prior to doing so, however, we need to introduce the role of regulatory capital restrictions, as all modern depository institutions are subject to them, and then incorporate the classical M&M methodology. This is carried out next.

4 The Regulatory Capital Restrictions

As a depository institution borrows and lends, it would very likely be subject to some tight government controls. These controls are known as regulatory capital and, basically, they require the establishment to maintain bounds on some of its ratios, most importantly the Tier 1 and Tier 2. There are, indeed, a number of other restrictions that apply as well (*Berger et al*, 1995), but we shall focus only on the Tier 1, as it happens to be, arguably, one of the most, if not the most, critical (*Schmittmann et al*, 1996).

The definition of the Tier 1 ratio, T_1, depends on interpretation. In this work, we shall keep it as plain as possible and simply define it as:

$$T_1 \equiv \frac{E}{RWA} \tag{4.1}$$

where E is the lender's equity and RWA is its total "risk-weighted assets". The idea here is that the lender must have sufficient equity to cover its losses in case the borrower defaults.

The next important issue involves determination of RWA. In simple terms, RWA is equal to the amount of loan exposure the institution has to its borrowers, multiplied by their individual "risk weight". Roughly speaking, therefore, if we were to consider a single borrower with a risk weight of r and that the lender is lending *all* its assets, $D + E$, to that single borrower, then the Tier 1 ratio would be

$$T_1 = \frac{E}{r(E+D)} = \frac{1}{r(1+\phi)} \tag{4.2}$$

where ϕ is the leverage.

Needless to say, different types of borrowers carry different risk weights. To get these, there are a variety of approaches available. These consist of either placing the borrower into certain categories, such as "sovereign," "bank" or "corporate," where the respective risk weight could be determined according to Table 1[18], or taking the more tenuous, but comprehensive, route to computing it given two parameters, namely, the probability of default after one year, P_D, and recovery ratio, R_R,[19] of the borrower. This relationship[20] is expressed by (*Ischenko and Samuels*, 2001):

$$r = \min \left[\frac{(1-R_R)}{50} \times 976.5 \times N(1.043 \times G(P_D) + 0.766) \times \left(1 + \frac{0.0470(1-P_D)}{P_D^{0.44}} \right), 12.5(1-R_R) \right] \tag{4.3}$$

whose behaviour is illustrated in Figure 5 as *r versus* P_D for variable R_R. Here, P_D is limited to a minimum of 0.03%, $N(*)$ is the normal cumulative distribution and $G(*)$ the inverse of the normal cumulative distribution, with mean 0 and standard deviation of 1. Consequently, given the borrower's P_D and R_R, the risk weight could be evaluated via Equation 4.3 [for a retailer], from which the risk-weighted assets, RWA, and the Tier 1 ratio, T_1, could ultimately be computed. Clearly, therefore, Equation 4.3 provides r in a more rigorous and continuous manner and, hence, we shall, from now on, follow this route instead of the one outlined in Table 1, to calculate the risk weights.

TABLE 1: RISK WEIGHTS UNDER THE "STANDARDIZED APPROACH." [REPRINTED, WITH PERMISSION, FROM *ISCHENKO AND SAMUELS* (2001)]

Type of Exposure	Basel II					
	AAA to AA-	A+ to A-	BBB+ to BBB-	BB+ to B-	Below B-	Unrated
Sovereigns	0	20	50	100	150	100
Banks (option 1 — based on sovereign's rating)	20	50	100	100	150	100
Banks (option 2 — based on bank's rating)	20	50 L-T* 20 S-T*	50 L-T* 20 S-T*	100 L-T* 50 S-T*	150	50 L-T* 20 S-T*
	AAA to AA-	A+ to A-	BBB+ to BB-		Below BB-	Unrated
Corporates	20	50	100		150	100

* L-T = original maturity of over three months, S-T = three months or less.

Sources: Basel Committee on Banking Supervision and Schroder Salomon Smith Barney.

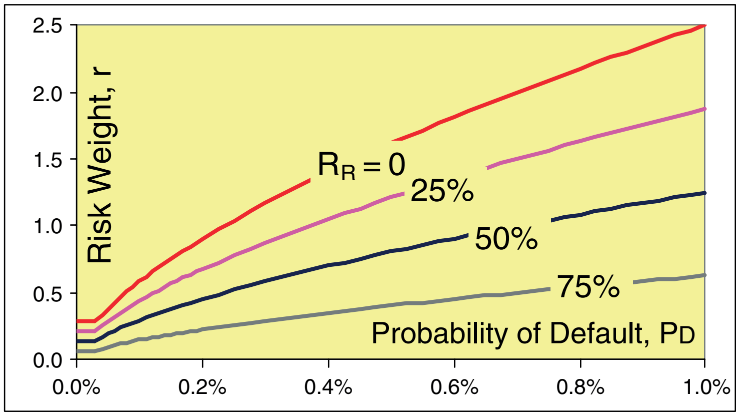

Figure 5: The risk weight, *r*, versus the probability of default, P_D, after one year. This is plotted according to Equation 3.4, with P_D limited to a minimum of 0.03%

5 The M&M Treatment: Accounting for Risk and Spread

To account for the credit spread arising from the risk of default, we generalise Equation 3.7 as

$$R_E = R_T(1-T) + (R_T - R_D)(1-T)\phi \tag{5.1}$$

where the removal of asterisks from R_T, R_D and ϕ reflects a risky environment. Also, to proceed, we need to implement certain assumptions on how R_T and R_D are affected by default risk. As these happen to be separate issues, the former related to the borrower's cost of debt and the latter to the lender's, we treat them independently in Sections 5.1a and 5.1b.

5.1a The impact of default risk on the borrower's cost of debt, R_T

The risk of default and, subsequently, credit spread of any borrower, borrowing from a lending institution at rate R_T, is generally characterised by two parameters: the probability of default, P_D, and the recovery ratio, R_R, both of which have been defined earlier. If, in the interest of simplicity, we limit both parameters to a one-year horizon and avoid the nonlinearities that come with continuous compounding[21], we could express the risk-neutral relationship between R_T^* and R_T as:

$$1 + R_T^* = (1 - P_D)(1 + R_T) + R_R P_D (1 + R_T)$$

which can be re-arranged as

$$R_T = \frac{R_T^* + (1 - R_R)P_D}{1 - (1 - R_R)P_D} \tag{5.2}$$

Note that the difference between R_T and R_T^* is the credit spread of the borrower, which remains positive as long as $0 \le P_D, R_R \le 1$. Accordingly, also, when either P_D is zero or R_R is unity, R_T becomes identical to its risk-less counterpart, R_T^*, an observation that is consistent with intuition.

5.1b The impact of default risk on the lender's cost of debt, R_D

For a lending institution as well, the cost of debt depends on the probability of default and recovery ratio. The approximate governing relationship between them is, in fact, identical to the one depicted in 5.2.

The critical issue here, notwithstanding, is determination of P_D and R_D. For this, a variety of credit models, such as Moody's and S&P's, are available. These models, basically, encompass several factors and ratios that are obtainable from the financial statement. Nevertheless, the commonality among all the different models is a strong dependence on leverage (*Constantinides et al*, 2001), which appears also to present itself, directly or indirectly, in Merton's model (*Merton*, 1974).[22] For this reason, therefore, we shall assume here that the lenders credit spread, Δ, which embodies the margin between the risky cost of debt, R_D, and its risk-free counterpart, R_D^*, is simply a function of the leverage, ϕ,i.e.

$$\Delta = \Delta(\phi) \tag{5.3a}$$

Furthermore, we shall assume, solely for the purposes of this work, that this relationship is expressible by:

$$\Delta = \left(\frac{\phi}{100}\right)^2 \tag{5.3b}$$

We should stress that Equation 5.3b, whose behaviour is plotted in Figure 6, is purely hypothetical and being employed here in this form

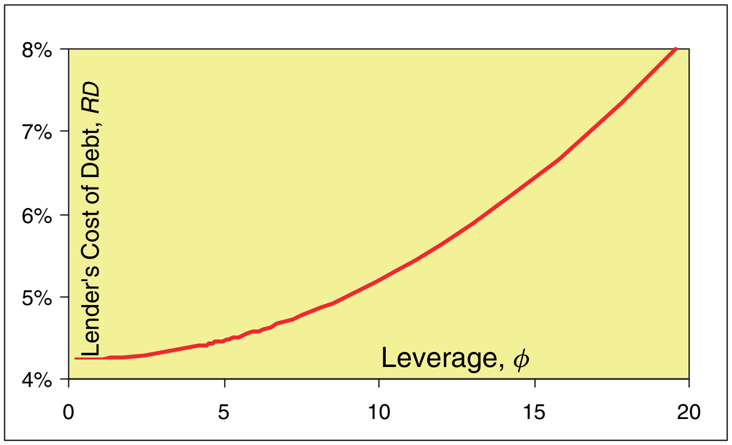

Figure 6: The lender's cost of debt, R_D, plotted verses leverage, ϕ, according to the presumed spread relationship given by Equation 5.3b. The risk-free cost of debt, R_D*, which coincides with ϕ = 0, is assumed here to be 4%

only for convenience—to enable us to carry on with the calculations that follow hereafter.

5.1c Impact of the risk of default on the enterprise value

It would now be useful to assess the major features that distinguish between the enterprise values of a risk-less and risky lender. We should recall that, in the case of corporate firms, these differences capture the essence of the optimal capital structure, whereby as a risk-less firm's value is devoid of a maximum [optimum], the value of risky firm could very well possess one (*Cohen*, 2001b).

In computing the risk-less and risky enterprise values of a depository institution, which we denote here by V^* and V, respectively, and define as

$$V^* = D^* + E \tag{5.4a}$$

and

$$V = D + E \tag{5.4b}$$

we refer to Table 2, where the procedure is explained in some depth beneath it. In the interest of space, we shall avoid going through the details here and, instead, provide Figure 7, which illustrates the comparison between V^* and V as a function of debt, D. It is noted here, in particular, that, in contrast to a corporate firm, the enterprise value, V, of a risky lender does not show any tendency to acquire a maximum at a finite debt. The reason for this is that the lender is maintaining a *certain* leverage as it moves along its fundamental constant and, hence, in the process of raising its debt, the leverage does not change.

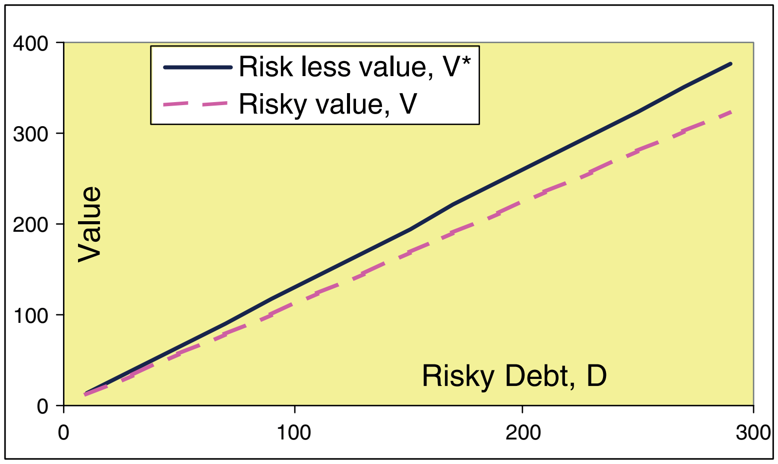

Figure 7: Comparison of risk-less and risky enterprise values, V^* and V, respectively, as a function of risky debt, D, while maintaining the leverage, which is the fundamental constant, uniform. The underlying numbers have been generated from Table 2.

TABLE 2: CALCULATING THE VALUE OF THE ENTERPRISE AS A FUNCTION OF DEBT, WHILE HOLDING THE LEVERAGE CONSTANT. THESE NUMBERS, WHICH ARE BASED ON ϕ^* = 10 AND R_D^* = 4%, UNDERLIE THE CURVES IN FIGURE 7

D (1)	R_D (2)	D* (3)	E (4)	ϕ(5)	R_D (6)	V* (7)	V (8)
10	4.72%	11.8	1.2	8.48	4.72%	13.0	11.2
30	4.72%	35.4	3.5	8.48	4.72%	38.9	33.5
50	4.72%	59.0	5.9	8.48	4.72%	64.9	55.9
70	4.72%	82.6	8.3	8.48	4.72%	90.8	78.3
90	4.72%	106.2	10.6	8.48	4.72%	116.8	100.6
110	4.72%	129.8	13.0	8.48	4.72%	142.7	123.0
130	4.72%	153.4	15.3	8.48	4.72%	168.7	145.3
150	4.72%	176.9	17.7	8.48	4.72%	194.6	167.7
170	4.72%	200.5	20.1	8.48	4.72%	220.6	190.1
190	4.72%	224.1	22.4	8.48	4.72%	246.5	212.4
210	4.72%	247.7	24.8	8.48	4.72%	272.5	234.8
230	4.72%	271.3	27.1	8.48	4.72%	298.5	257.1
250	4.72%	294.9	29.5	8.48	4.72%	324.4	279.5
270	4.72%	318.5	31.9	8.48	4.72%	350.4	301.9
290	4.72%	342.1	34.2	8.48	4.72%	376.3	324.2

(1) Risky debt, starting at 10 and raised at increments of 20.
(2) Risky cost of debt of the lender. Value fed from Column 6.
(3) Risk less debt, calculated from Footnote 7.
(4) Equity, calculated from holding the risk less leverage, ϕ^*, constant at 10.
(5) Risky leverage calculated as D/E.
(6) Risky cost of debt of the lender calculated from Equation 5.3b, with R_D^* = 4%.
(7) Risk less enterprise value, calculated as V* = D* + E.
(8) Risky enterprise value, calculated as V = D + E.

The procedure needed to derive V^* and V *across* different values of leverage is somewhat more complicated and, hence, we have decided to omit it from here. The final outcome, however, is summarised in Figure 8, which is based on the "realistic-case" scenario outlined in a following section [i.e. see Section 5.2d]. Once again, we note the absence of a maximum in the enterprise value in either of the cases, V and V^*. This, therefore, justifies the need for another concept to define the optimal capital structure of a depository institution.

5.2 Impact of variable R_T and R_D on the return on equity, R_E

The outcome of the preceding sections could now be categorised into four distinct scenarios. These are (i) an ideal case, where both R_T and R_D remain constant, independent of the lender's leverage, (ii) a "semi-ideal" situation, with variable R_T and constant R_D, (iii) another semi-ideal situation, in which R_T remains constant, but R_D varies and,

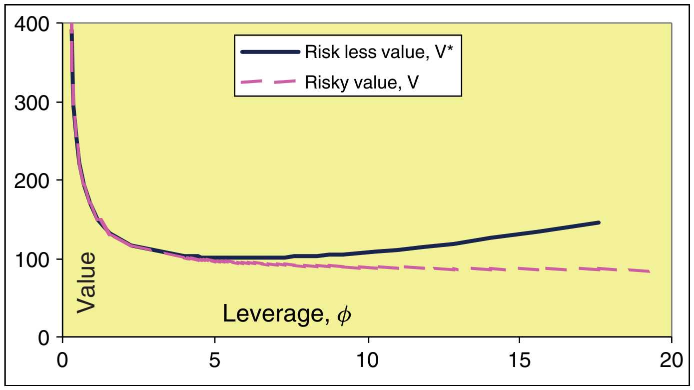

Figure 8: Comparison of risk-less and risky enterprise values, V^* and V, respectively, as functions of risky debt, D, across variable leverage. Note the absence of a maximum in the case of V, which contrasts to the case of a risky corporate firm, where a maximum defines the optimal capital structure

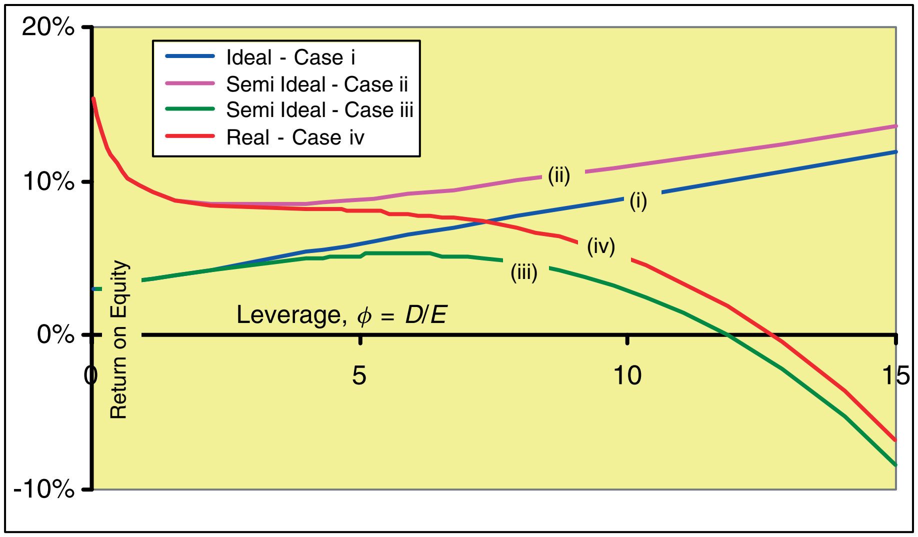

Figure 9: The return on equity, R_E, plotted against leverage, ϕ, for the different cases described in Sections 5.2a–d—i.e. Cases i-iv, respectively. These cases are tabulated in Table 3

finally, (iv) a realistic scenario, where both, R_T and R_D, vary according to leverage. For convenience, the above cases, along with some numerical values, are tabulated in Table 3 and by a demonstration next.

5.2a [Case i] The Ideal Case: Both R_T and R_D Constant

Evidently, the ideal-case scenario, in which both R_T and R_D remain constant, happens to be also the simplest. Here, as in all the other cases, Equation 5.1 is employed to obtain the impact of leverage, ϕ, on R_E, as we hold R_T, R_D, and T constant at 5%, 4% and 40%, respectively. The result of this is displayed in Figure 9, where, as revealed earlier, we find that R_E increases linearly with ϕ.

5.2b [Case ii] The Semi-ideal Case: Variable R_T and Constant R_D

This case represents a risk-less lender, who carries a constant cost of debt, i.e. R_D^*, lending to a risky borrower, whose borrowing rate, R_T, varies with its default probability and recovery ratio, P_D and R_R, respectively, all in line with Equation 5.2. The two regimes, which present themselves in Figure 9, are one of falling R_E, occurring at lower values of leverage and one of rising R_E, happening at higher leverage.

The initial regime of declining R_E evolves from the complex interactions between P_D, R_R, ϕ and r, all coming from Equations 4.2 and 4.3 combined. Here, one observes that the variation in ϕ results from changing the borrower's risk weight, r, as per Equation 4.2 [upon holding the Tier 1 ratio constant at 8%], while the change in r, is brought on by varying the borrower's P_D and R_R.

Therefore, at low values of ϕ, where the borrower's risk weight is high, the credit spread of the borrower becomes comparatively large. This, thereby, translates into a high R_T, which, consequently, raises the level of R_E in accordance with Equation 5.1. Likewise, upon increasing ϕ in Figure 9, one should, again, expect a rise in R_E, as reflected by Equation 5.1.

Altogether, this situation approaches reality when the lender has a tight safety net, such as the government, to absorb the risk. This, therefore, creates a strong tendency for the lender to operate at high leverage, whereby the return on equity is maximised and, at the same time, the risk on equity capital is minimised.

TABLE 3: DESCRIPTION OF THE DIFFERENT CASES UNDERLYING SECTIONS 5.2A-D AND FIGURE 9. IN ALL CASES THE TAX RATE, T, AND THE TIER 1 RATIO, T_1, WERE HELD CONSTANT AT 40% AND 8%, RESPECTIVELY

Case	R_T	R_D
i	Constant (at 5%)	Constant (at 4%)
ii	Constant (at 5%)	Variable
iii	Variable	Constant (at 4%)
iv	Variable	Variable

5.2c [Case iii] The Semi-ideal Case: Constant R_T and Variable R_D

In this instance, the lender suffers from the risk of default as it levers up, whereas the borrower remains risk-less and maintains a constant borrowing rate throughout, equal to R_T^*. The lender's credit spread, meanwhile, is represented here by the simple functional relation given by Equations 5.3a–b. It should be emphasised that, although this relationship is purely hypothetical and being employed here for reasons no other than convenience and simplicity, its outcome should constitute a qualitative representative of more complicated situations.

In view of the above and in reference to Figure 9, we observe that R_E first rises and then falls with leverage. This characteristic, therefore, truly illustrates an "optimal capital structure" in the sense that a unique balance between debt and equity does exist, leading to a maximum in the return on equity. Note that this is different from a corporate firm's optimal capital structure, where one seeks to maximise the value of the firm (Cohen, 2001b).

5.2d [Case iv] The realistic case: both R_T and R_D variable

This final scenario, in which both R_T and R_D are variable, depending on the leverage of both, the lender and the borrower, portrays the most realistic of all the ones considered so far. In fact, the outcome here is a result of the complex interactions between lender and borrower, as depicted by Equations 4.2–4.3 and 5.1–5.3.

In this instance, the risk weight on the loan made to the borrower[23] determines the leverage that the lender must comply with in order to satisfy the limit on the Tier 1 ratio. This leverage, on the other hand, would impact the lender's credit rating and, hence, its cost of debt. The above-mentioned interaction, therefore, leads to the fourth curve in Figure 9, where we note that the return on equity falls as leverage goes up. This behaviour contrasts sharply with the initial scenario, where R_E rises with leverage. As a result of this directional change, the lending institution may not find it suitable to operate at high values of leverage as it would have favoured in the previous cases, i and ii.

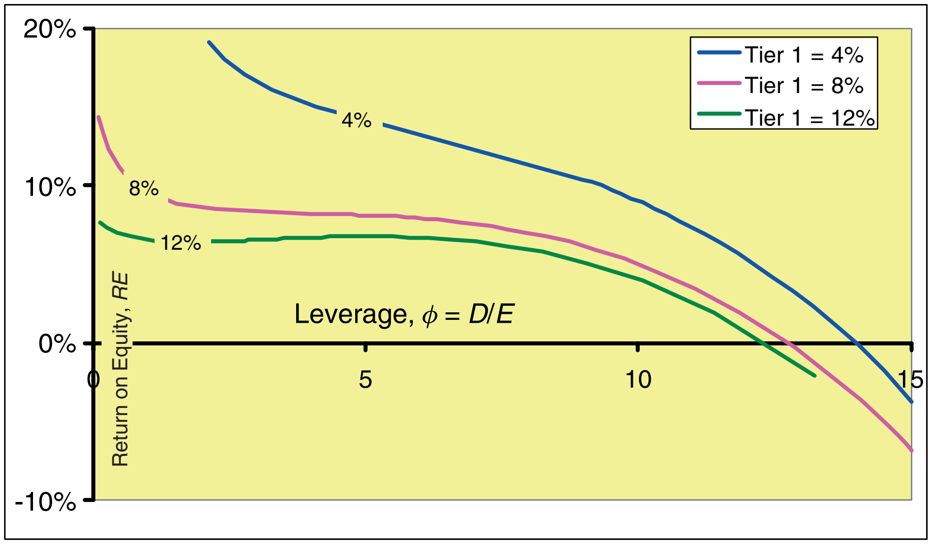

Figure 10: The impact of the Tier 1 ratio on the realistic scenario portrayed in Figure 7. Note the contraction in the regime of positive return on equity with increasing T_1

We have, for interest, also included Figure 10, which portrays the impact of the Tier 1 ratio on R_E, centring on the realistic case outlined in Figure 9. It is observed that, at least within the range of T_1 considered here, the above-mentioned reversal in behaviour pattern, going from ideal to realistic, is present.

Another important point is the absence of the maximum return on equity, R_E. This basically suggests that, in realistic cases where both R_T and R_D vary with leverage, looking for an optimal capital structure, at least within the context of maximising R_E, would be futile. It, thus, follows that for a depository institution, one must find alternative measures to define and pinpoint the optimal capital structure.

6 Determining the Optimal Capital Structure for a Depository Institution

As concluded above, pinning down the optimal capital structure for a depository institution is not so straightforward. The existence of such a regime, however, may still be justified if we were to consider the following argument. Suppose that a lender intends to lend funds to a very low-risk borrower. At the same time, assume also that the lender is restricted to maintain a Tier 1 ratio, T_1, at no more than 8%. The combination, therefore, of the low-risk borrower, who has a risk weight, r, approaching zero, and the upper constraint of 8% set on the lender's T_1, would entice the latter to raise its level of debt to as high a leverage as possible, i.e. $\phi \to \infty$, as predicated by Equation 4.2. Acquiring such high leverage is, clearly, unacceptable, as, not only the return on equity of the lender falls quickly to negative domain [see Figure 8], but also the balance sheet becomes tarnished in the eyes of stakeholders[24] and the credit rating is put at risk[25].

The lender has another choice, nonetheless, which is simply to maximise its return on equity. This, according to the realistic scenario illustrated in Figures 9 and 10, occurs at *zero* leverage, which means that the lender is risking nothing else except its own equity. Thus, in case the borrower defaults on his debt, the lender has all to lose. This, also, is not practical.

So, what is the solution? A possibility is that the lender should seek a borrower whose risk-weight profile is compatible with the acceptable level of leverage the lender's balance sheet could assume, without arousing any suspicions from both, the stakeholders and the rating agencies. It is, therefore, this type of inter-dependence between the lender and the borrower that leads to an optimal capital structure for a lending institution, and our objective here is to quantify it.

6.1 A possible solution for the optimal capital structure

We refer once more to the realistic-case scenario in Figures 9 and 10, where, clearly, there does not appear to be a sensible, clear-cut sign of a maximum in the return on equity.[26] Hence,

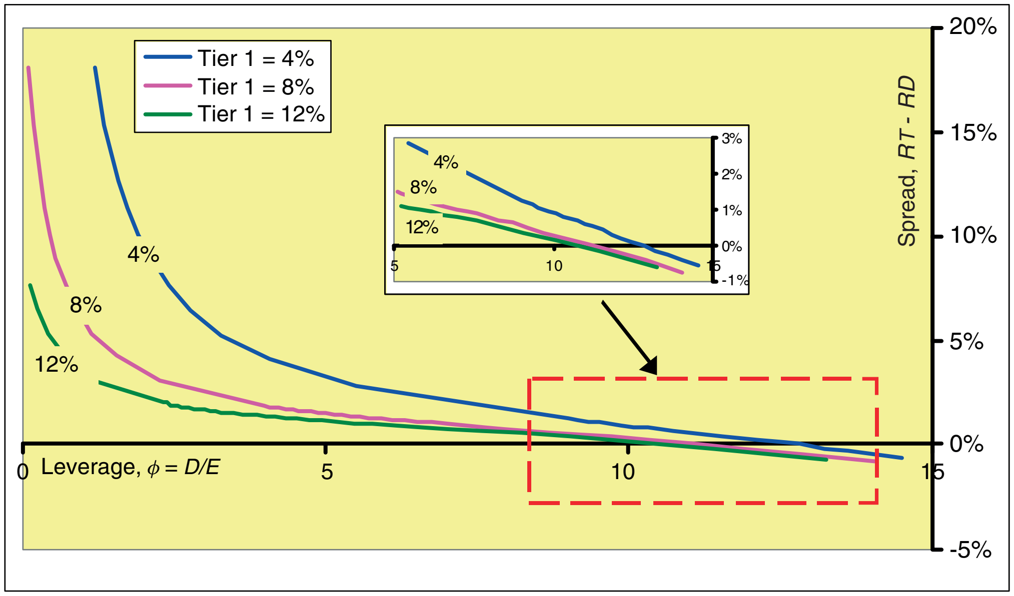

Figure 11: The impact of the Tier 1 ratio, T_1, on the spead $R_T - R_D$. The significance of this is that a spread satisfying the criterion $R_T - R_D \geq 0$ guarantees a positive return on equity [see Equation 5.1]

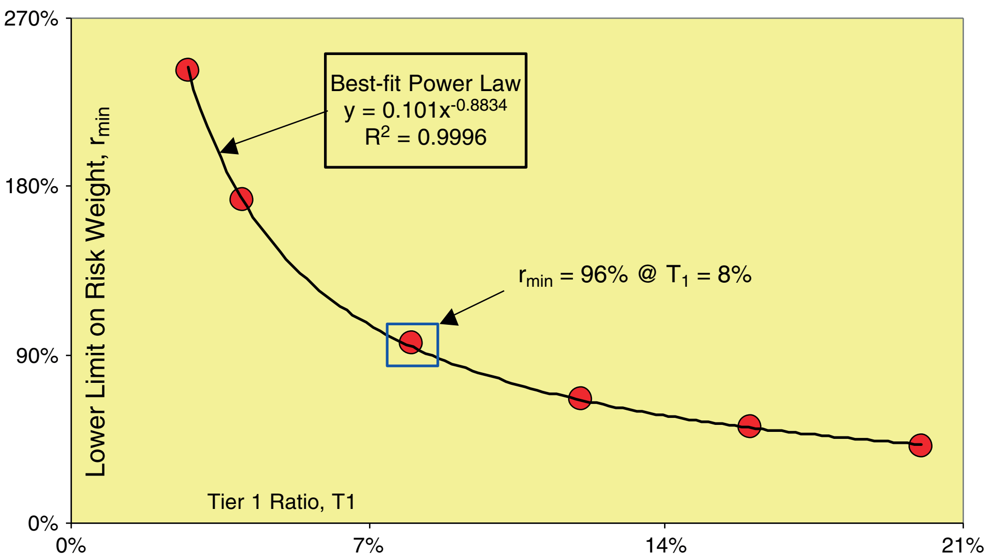

Figure 12: The impact of the Tier 1 ratio, T_1, on the minimum risk weight, r_{min}. The point highlighted is of T_1 = 8%, which corresponds to an r_{min} of 96%. Together with Equation 4.2, therefore, this sets a maximum limit on the leverage, ϕ, at 12. This means that a lender with a constrained Tier 1 ratio is restricted to a certain risk exposure and leverage if it were to maintain a positive return on equity. Finally, for sake of curiosity, the best-fit power-law curve is also included, which, interestingly, has an R^2 greater than 99.9%

we must resort to a different type of rationale to support the notion of an optimal capital structure, namely one that follows the argument in the preceding section. To achieve this, we begin with the assumption that the lender's primary objective is to deliver to its shareholders a *positive* return on equity. Subsequent to Equation 5.1, therefore, we note that this is achievable if and only if the margin between R_T and R_D satisfies the following condition:

$$R_T - R_D \geq 0 \tag{6.1}$$

from which results similar to the ones in Figure 11 could be derived. Following along this line of reasoning, therefore, where the optimal capital structure is being related to the interaction between the Tier 1 ratio and the above criterion, we propose next an alternative, but logical, framework for defining an optimal capital structure for a simple depository institution.

6.2 Framework and result

An important by-product of Figure 11 is the impact of the Tier 1 ratio, T_1,on the point of leverage, ϕ, where the margin $R_T - R_D$ crosses the boundary from positive to negative. This leverage could be viewed as a "threshold" invoked by the constraint on T_1. Hence, an institution whose Tier 1 ratio is capped cannot surpass a certain threshold leverage, since this leads to a breakdown of the criterion set by Equation 6.1. Moreover, given that ϕ is related to the risk weight, r, via Equation 4.2, one could, subsequently, obtain a mapping of the risk weights that a lending institution could undertake safely in its portfolio of loan exposures.

Applying the above logic to Figure 11, therefore, leads to the type of outcome illustrated in Figure 12, where, for instance, a minimum limit of 8% enforced on the Tier 1 ratio, in addition to the observation that the quantity $R_T - R_D$ crosses the boundary at a ϕ of approximately 12%, would impose a lower bound on the borrower's risk weight, limiting r to a minimum, i.e. r_{min}, of 96%. It thus follows from this [constraint on the risk weight] that one could establish, from either Table 1 or Figure 5, precisely the type of firm, in terms of default probability and recovery ratio, the bank should target as a potential borrower.

7 Summary and Conclusions

Application of capital structuring to depository institutions was the focus of our attention here. Although the nature of the problem is, inarguably, complicated, we tried to simplify it here, mainly to illustrate how the lender and borrower interact. In doing so, we arrived at several conclusions, some of which are:

1. The basic M&M principles are applicable to depository institutions as well, but only after accounting for the fundamental differences in how lenders and corporates operate. It is demonstrated that, unlike the application of M&M to a corporate firm, where one could pin point an optimal capital structure in terms of the firm's value, there does not appear to be a clear-cut answer when it comes to a lending institution. This, very simply, is due to the nature of the two businesses, namely in the way their revenues are generated. Furthermore, regulations imposed on capital can restrict the type of borrower the bank could lend to. This interaction between the bank and the borrower significantly complicates the matter.
2. The fundamental constant for a lending institution, as derived from the M&M methodology, turns out to be the leverage. This contrasts sharply to that of a corporate firm's, for which the constant is the unlevered value.
3. The relationship between the levered and unlevered betas of a lending institution also happens to be different from that of corporate firm's. The cause of this is, once again, the variations in way the two types of establishments derive their revenues.
4. In comparison to a corporate firm, there appears to be no well-defined notion of an optimal capital structure for a depository institution. There are two reasons for this. Firstly, if we were to consider the value of the firm to be the determining factor, such an optimal would be, for all intents and purposes, absent. As proof, we refer to Figure 8, where the value, V, is plotted against leverage, ϕ. We observe here that V is maximised at zero leverage, which is not a feasible solution, owing to reasons stated earlier.

   Secondly, if we relied on maximising the return on equity to define the optimal capital structure, we observe, once more, that there is a lack of a practical optimum, particularly in Case iv of Figure 9, which represents the realistic scenario with both, the lender and borrower, vulnerable to default risk. As a result, therefore, the idea of defining an optimal capital structure for a depository institution—one that is similar in concept to a corporate firm's—remains ambiguous and, hence, dependent on subjective inputs.
5. If we were to focus instead on the margin, $R_T - R_D$ [and its relation to maintaining a positive return on equity], as essential to supporting the notion of an optimal capital structure, we arrive at the results in Figures 11 and 12. These summarise the interactions between the lender and borrower, which were described earlier, by highlighting the connection between the capital regulation imposed on the lender, in terms of Tier 1 ratio, and the risk exposure of the lender to the borrower.

What we have accomplished so far, in our attempt to investigate the capital structure of a simple lending institution, was merely "touch the tip of the iceberg." This, notwithstanding, was sufficient to verify that analysing the capital structure of banks is, to say the least, very complicated. Never the less, a simplistic approach, following along these lines could, possibly, present a viable route to locating the optimal capital structure.

FOOTNOTES & REFERENCES

1. March 2003. Paper can be found in *http://rdcohen.50megs.com/DepInstabstract.htm*.
2. I express these views as an individual, not as a representative of companies with which I am connected.
3. E-mail: *ruben.cohen@citigroup.com*—Phone: +44(0)207 986 4645.
4. A depository institution is a type of financial institution that, in its simplest form, borrows funds from deposits and/or other establishments and lends them to borrowers. In this process, revenue is created mainly from the margin between the rate of lending and the cost of borrowing.
5. See, for instance, *Cohen* (2001a,b) for similar derivations of the M&M principles, with and without the impact of credit spread.
6. EBIT stands for *e*arnings *b*efore *i*nterest and *t*ax.
7. By "risk-less" debt we mean the value of debt if the firm were fully immune to default risk. In this case the cost of debt, R_D, would be equal to a constant, independent of leverage. The relationship between D^* and D is simply $D^* = R_D D / R_D^*$ (Cohen, 2001b).
8. V_u and R_u remain constant under the assumption of *no* default risk and credit spread. Equation 2.2 is, subsequently, not valid when these are present.
9. For the financial statement in Figure 1 , V_u is calculated as 110, which gives an R_u of $20 \times (1 - 40\%)/110 = 10.9\%$.
10. In computing the levered value from V_u, the method for dealing with risky and risk less firms is different. See *Cohen* (2001a,b) for an illustration.
11. Lending could, in this case, be also achieved via the purchase of bonds.
12. The asterisk in R_T and R_D indicates that they are constant.
13. After other expenses, including SG&A, depreciation and amortisation.
14. The left-hand side of Equation 3.4 is, in fact, the weighted average cost of capital
15. The significance of moving along this constant is that the enterprise value computed from the income statement is guaranteed to be consistent with that calculated from the balance sheet.
16. Recall that $\beta_L = \beta_u[1 + (1\text{-}T)\phi^*]$for a corporate firm (*Cohen*, 2001b).
17. For instance, this could occur at high tax rates.
18. See also Page 151 in *Schroeck* (2002) for another display.
19. Many sources use the "loss given default," or *LGD*, instead of the recovery ratio. The relation between the two is $LGD = 1 - R_R$.
20. Equation 4.3 is for "retail." For "non-retail," we use 1.118 and 1.288, respectively, in place of 1.043 and 0.766.
21. The linearised form given here is a satisfactory approximation to the continuous compounding alternative as long as $R_T << 1$, which is generally the case.
22. Volatility is also an important element, however it shall be ignored here purely for sake of simplicity.
23. Recall that the borrower's risk weight depends on its default probability and recovery ratio [see Equation 4.3].
24. Stakeholders constitute depositors, bondholders, shareholders and regulators (*Schroeck*, 2002).
25. Recall from Section 5 that the default risk and, thus, the credit spread of the lender depends on its leverage, among other factors. The significance of leverage on credit rating is owed especially to interest rate risk [see, for instance, *Fabozzi* (1999) for a discussion].

26. In theory, the optimal capital structure for this case, if we were to opt for maximum return on equity, turns out to be at zero leverage, which, clearly, is not practical.

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